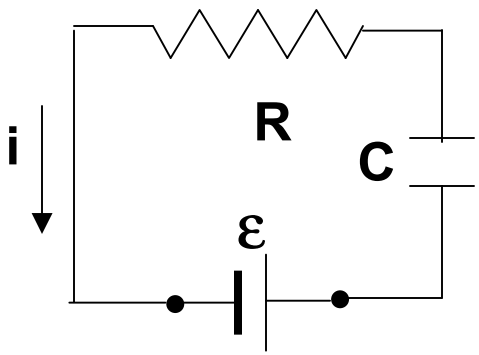
Circuito RC alimentado con cc

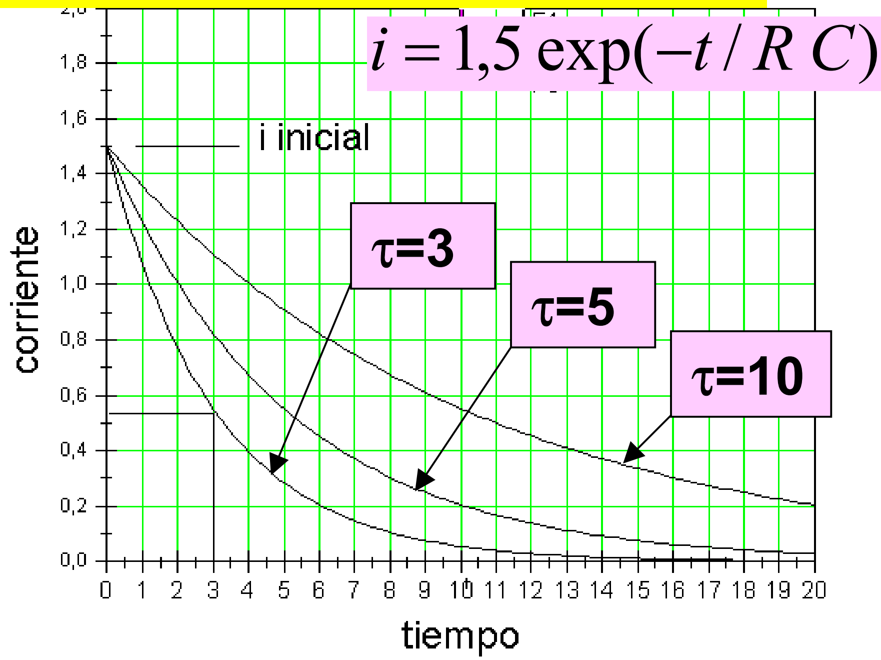


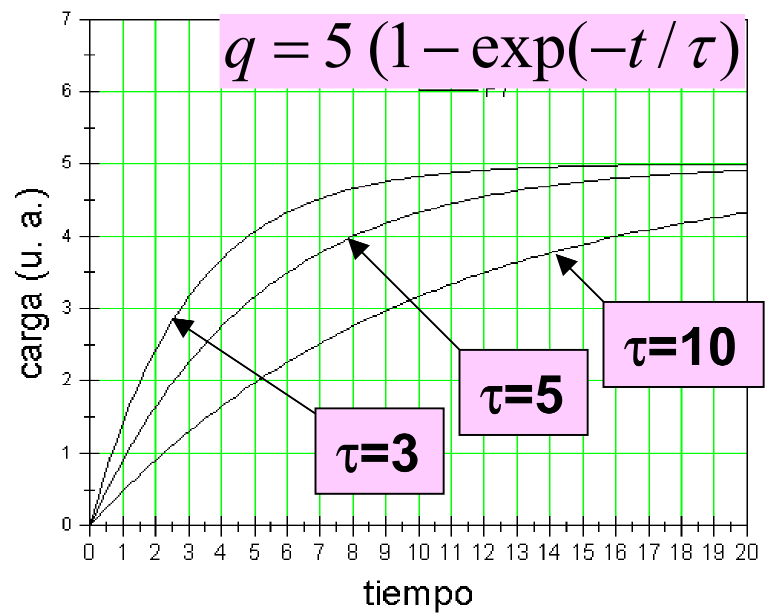
$$\varepsilon = i\,R + \frac{q}{C} = R\frac{dq}{dt} + \frac{q}{C} \qquad q = 0 \;\; en \;\; t = 0 \Rightarrow i_0 = \frac{\varepsilon}{R}$$

$$R\frac{di}{dt} + \frac{i}{C} = 0 \qquad \frac{di}{i} = -\frac{1}{R\,C}dt \qquad i = \frac{\varepsilon}{R}\,e^{-\frac{t}{R\,C}}$$

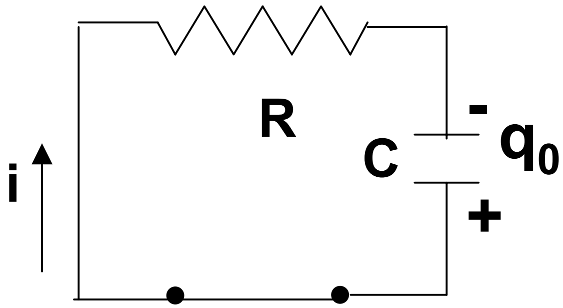
$$q = \int_0^t i\,dt = \frac{\varepsilon}{R}(-R\,C)\,e^{-\frac{t}{R\,C}}\Bigg)_0^t \qquad q = \varepsilon\,C\left(1 - e^{-\frac{t}{R\,C}}\right)$$

τ=RC: constante de tiempo RC





Curva de descarga del capacitor

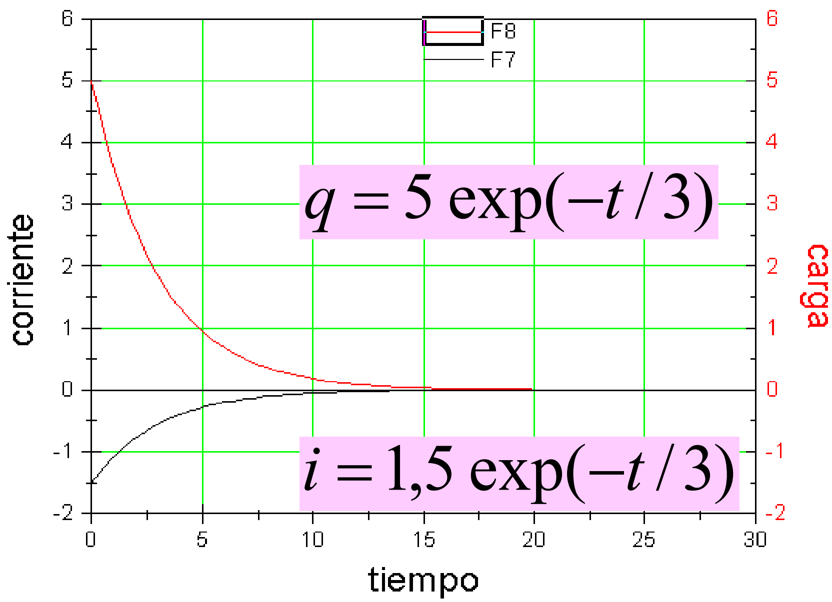


En determinado momento de la curva de carga se cortocicuita la fem, por ej. cuando $q=q_0$

$$\frac{q}{C} + i' R = \frac{q}{C} + \frac{dq}{dt} R = 0$$

$$\frac{dq}{q} = -\frac{dt}{R\,C}$$

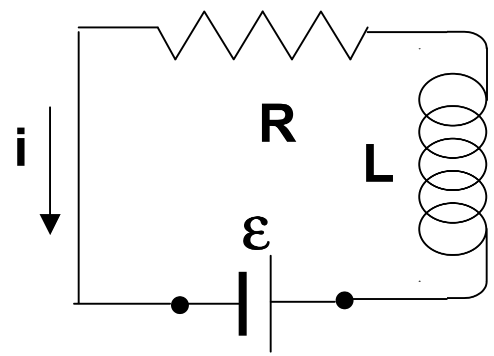
$$q = q_0\, e^{-\frac{t}{R\,C}}$$



$$i = \frac{dq}{dt} = -\frac{q_0}{R\,C}\, e^{-\frac{t}{R\,C}}$$

i en sentido contrario

Circuito RL alimentado con cc



$$\varepsilon = i\,R + L\,\frac{di}{dt} \qquad R\,\frac{di}{dt} + L\,\frac{d^2 i}{dt^2} = 0 \qquad s = \frac{di}{dt}$$

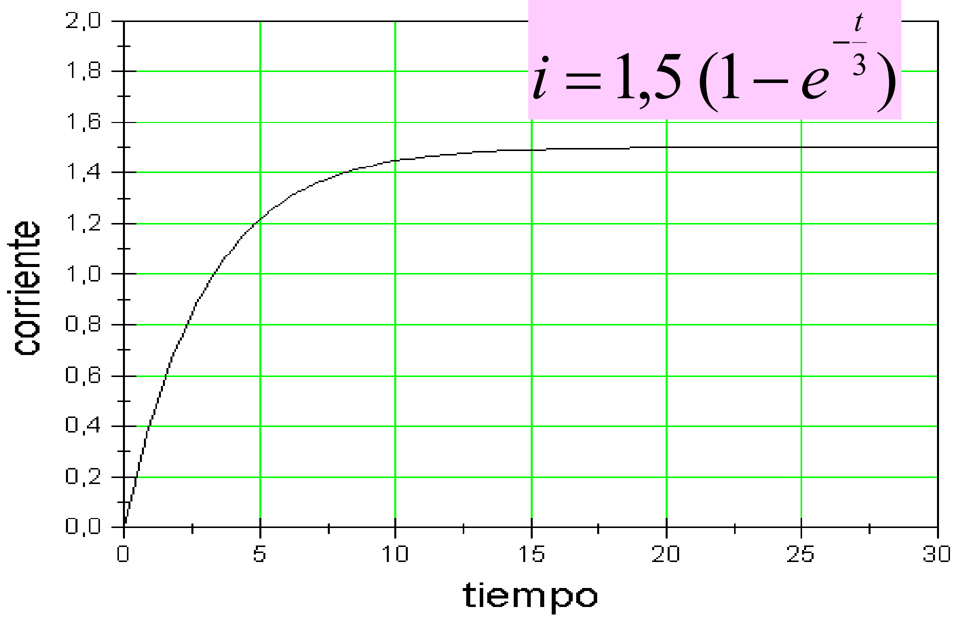
$$R\,s + L\,\frac{ds}{dt} = 0 \qquad \frac{ds}{s} = -\frac{R}{L}\,dt \qquad \ln\frac{s}{s_{t=0}} = -\frac{R}{L}\,t$$

En t=0, i=o y $(di/dt)_{t=0} = s_{t=o} = \varepsilon / L$

$$s = \frac{\varepsilon}{L}\,e^{-\frac{R\,t}{L}} \qquad i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{R\,t}{L}}\right)$$

τ=L/R: constante de tiempo RL

$$i = 1{,}5\,(1 - e^{-\frac{t}{3}})$$



Si en t' se cc la fem

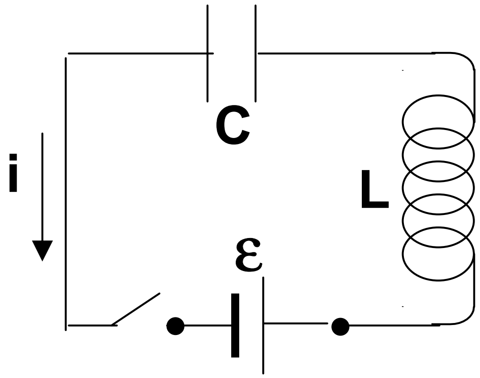
$$i\;R + L\,\frac{di}{dt} = 0 \qquad i = i_0\;e^{-\frac{R\,t}{L}}$$

Con CC

t = 0	**C: cc**	**L: ca**
t = ∞	**C: ca**	**L: cc**

Circuito LC alimentado con cc

$$t=0 \Rightarrow q=0 \; y \;\; i=0$$



$$\varepsilon - L\frac{di}{dt} - \frac{q}{C} = 0$$

$$L\frac{d^2 i}{dt^2} + \frac{1}{C} i = 0$$

Oscilador armónico simple

$$i = i_0 \cos(\omega t + \varphi)$$

$$\omega = \sqrt{\frac{1}{LC}} = 2\pi f = \frac{2\pi}{\tau}$$

i_0: amplitud; f: frecuencia natural circ. LC; φ: ángulo de fase; τ: período

$$t=0 \quad i=0 \Rightarrow i = i_0 \cos\varphi = 0 \Rightarrow \varphi = \pm\frac{\pi}{2}$$

En t=0, C es un cc y solo actúa L=> i(t=0)=0

$$t=0 \quad q=0 \Rightarrow \varepsilon = L\left(\frac{di}{dt}\right)_{t=0} = -i_0\, \omega L\, sen\,\varphi \Rightarrow i_0 = -\frac{\varepsilon}{\omega L} sen^{-1}\varphi$$

En t=o todo ε en L; C descargado

$$si \;\; \varphi = \frac{\pi}{2} \Rightarrow i_0 \langle \; 0 \Rightarrow \varphi = -\frac{\pi}{2}$$

$$i = \frac{\varepsilon}{\omega L}\cos\left(\omega t - \frac{\pi}{2}\right) = \frac{\varepsilon}{\omega L} sen\, \omega t$$

$$i_0 = \frac{\varepsilon}{\omega L} = \varepsilon\sqrt{\frac{C}{L}}$$

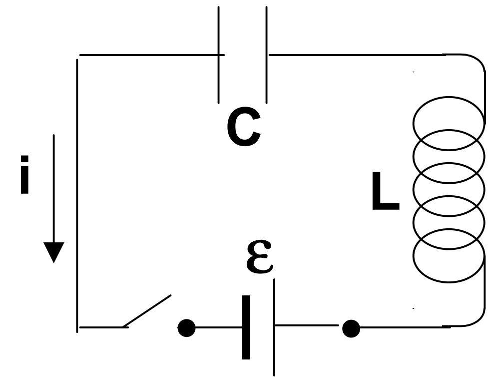
$$i = \frac{\varepsilon}{\omega L} \operatorname{sen} \omega t$$

Carga en el condensador

$$q = \int_0^t i \, dt = \int_0^t \frac{\varepsilon}{\omega L} \operatorname{sen} \omega t = \frac{\varepsilon}{\omega^2 L} (1 - \cos \omega t) = \varepsilon C (1 - \cos \omega t)$$

Caída de potencial en el condensador

$$\Delta V_C = \frac{q}{C} = \varepsilon (1 - \cos \omega t)$$

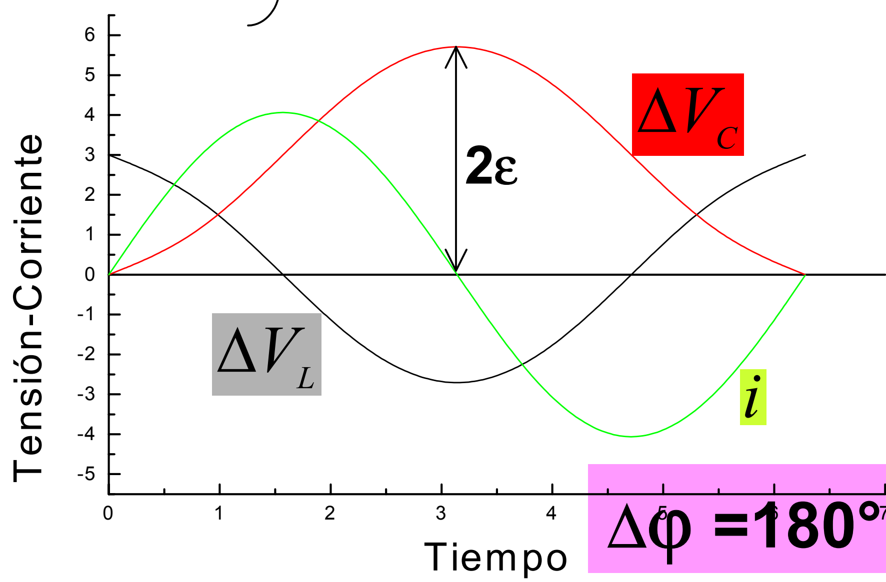


Caída de potencial en el inductor

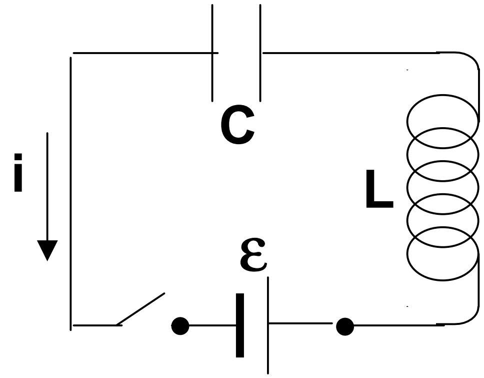
$$\Delta V_L = L \frac{di}{dt} = \varepsilon \cos \omega t$$

$$\Delta V_C + \Delta V_L = \varepsilon$$

Suma de tensiones parciales instantáneas en cada elemento =ε, <u>pero no de amplitudes</u> pues no tienen la misma fase



$\Delta\varphi$ **=180°**



$$i = \frac{\varepsilon}{\omega L} \operatorname{sen} \omega t$$

$$\Delta V_C = \frac{q}{C} = \varepsilon\,(1 - \cos \omega t)$$

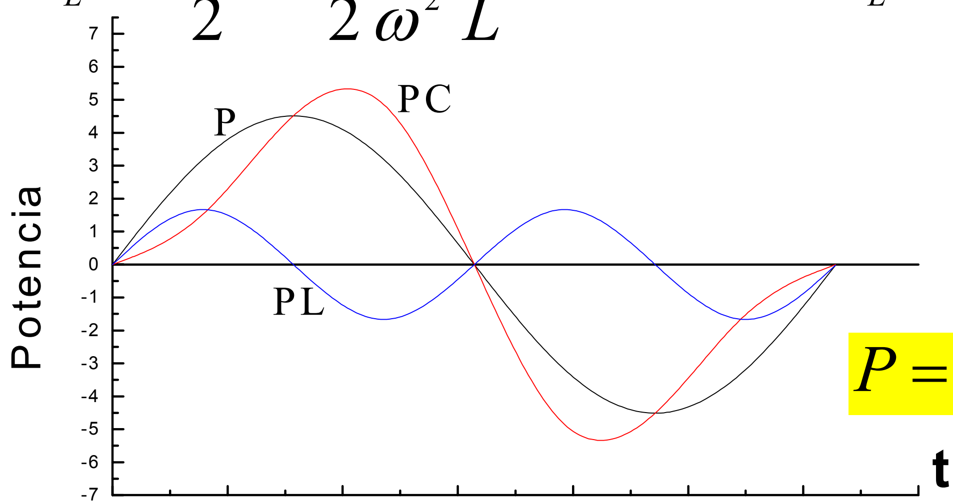
$$\Delta V_L = L \frac{di}{dt} = \varepsilon \cos \omega t$$

$$q = \varepsilon\, C\,(1 - \cos \omega t)$$

Potencia suministrada $P = i\,\varepsilon = \frac{\varepsilon^2}{\omega L} \operatorname{sen} \omega t = \varepsilon^2 \sqrt{\frac{C}{L}} \operatorname{sen} \omega t$

$$U_C = \frac{q^2}{2C} = \frac{\varepsilon^2 C}{2} (1 - \cos \omega t)^2 \qquad P_C = \frac{dU_C}{dt} = \frac{\varepsilon^2 C}{2}\, 2\,(1 - \cos \omega t)\, \omega \operatorname{sen} \omega t$$

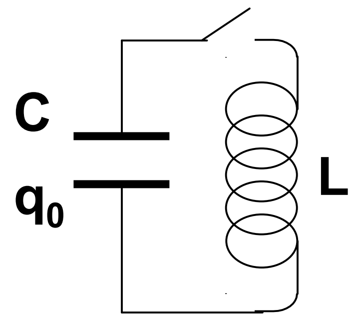
$$U_L = \frac{L i^2}{2} = \frac{\varepsilon^2}{2 \omega^2 L} \operatorname{sen}^2 \omega t \qquad P_L = \frac{dU_L}{dt} = \frac{\varepsilon^2}{2 \omega L}\, 2\, \omega \operatorname{sen} \omega t \cos \omega t$$



$$P_C = \varepsilon^2 \sqrt{\frac{C}{L}}\, (1 - \cos \omega t) \operatorname{sen} \omega t$$

$$P = P_C + P_L$$

$$P_L = \varepsilon^2 \sqrt{\frac{C}{L}} \operatorname{sen} \omega t \cos \omega t$$



$$L\frac{di}{dt}+\frac{q}{C}=0 \Rightarrow L\frac{d^2q}{dt^2}+\frac{q}{C}=0$$

$$q = q_0 \cos(\omega t+\varphi)$$

$$\omega = 1/\sqrt{LC}$$

$$Si\ \ t=0,\ q=q_0 \Rightarrow \varphi = 0$$

$$q = q_0 \cos \omega t$$

$$\Delta V_L = L\frac{di}{dt} = -\frac{q_0}{C}\cos\omega t = \frac{q_0}{C}\cos(\omega t-\pi)$$

$$i = \frac{dq}{dt} = -\omega\, q_0 \,\mathrm{sen}\, \omega t$$

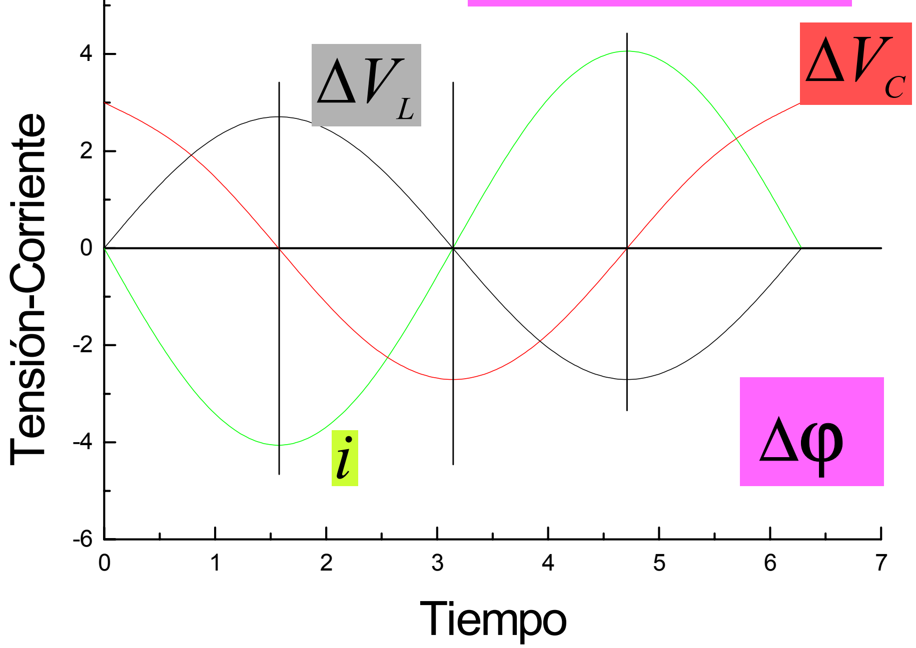
$$\Delta V_C = \frac{q}{C} = \frac{q_0}{C}\cos\omega t$$

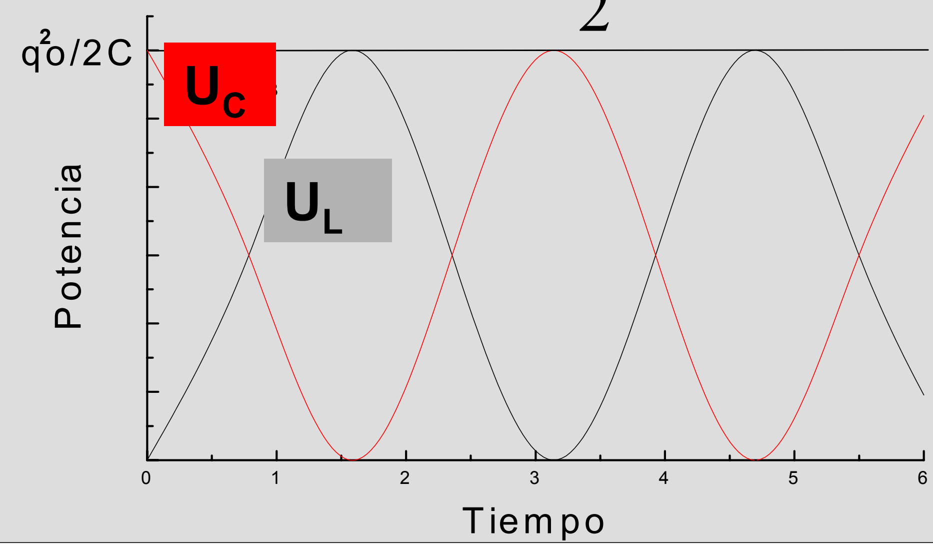
$$U = U_C + U_L = \frac{q_0^2}{2C} = \frac{q^2}{2C}+\frac{L i^2}{2}$$

$$U_C = \frac{q_0^2}{2C}\cos^2\omega t$$

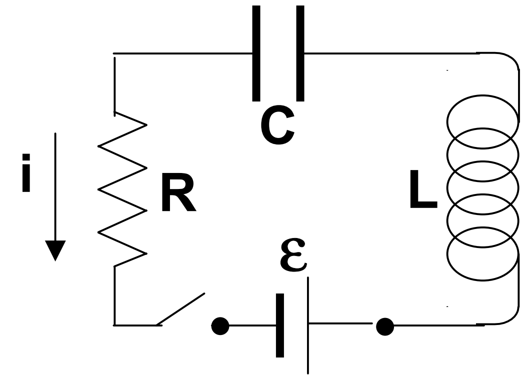
$$U_L = \frac{L\omega^2 q_0^2}{2}\mathrm{sen}^2\,\omega t$$

$$\Delta V_C + \Delta V_L = 0$$





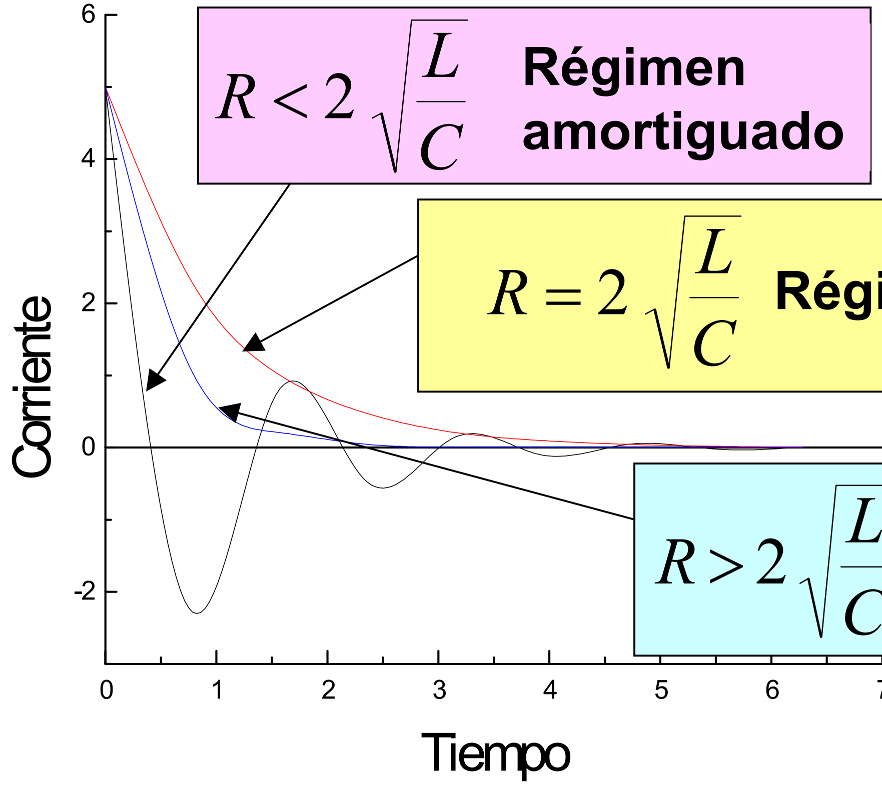
Circuito RLC alimentado por cc



$$\varepsilon = L\frac{di}{dt} + R\,i + \frac{q}{C} \qquad 0 = L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i}{C}$$

$$i = i_0 \exp\left(-\frac{R}{2L}t\right)\cos\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\,t + \varphi\right)$$

Solución solo válida para R chicas tal que $\frac{R^2}{4L^2} < \frac{1}{LC}$



$R < 2\sqrt{\frac{L}{C}}$ **Régimen amortiguado**

$R = 2\sqrt{\frac{L}{C}}$ **Régimen crítico**

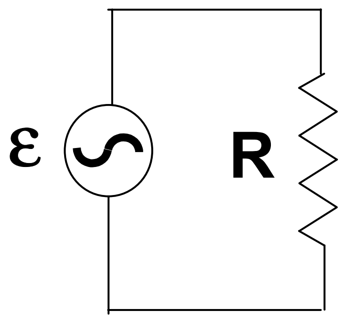
$$i = i_0 \exp\left(-\frac{R}{2L}t\right)$$

$$\tau = \frac{2L}{R}$$

$R > 2\sqrt{\frac{L}{C}}$ **Régimen sobreamortiguado**

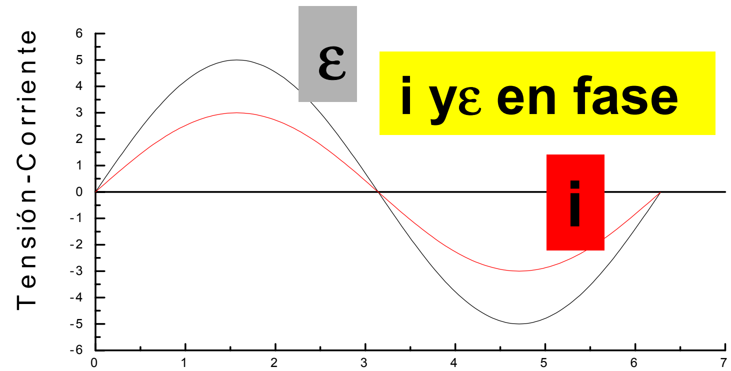
R, C y L alimentadas con ca

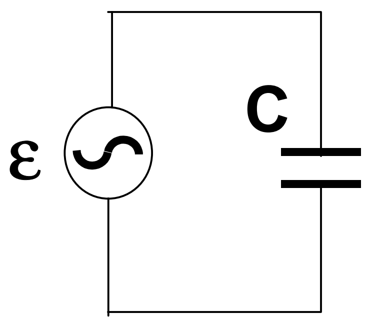
$$\varepsilon = \varepsilon_0 \ \text{sen}\ \omega t$$



$$i = \frac{\varepsilon}{R} = \frac{\varepsilon_0}{R}\ \text{sen}\ \omega t$$

$$i = i_0\ \text{sen}\ \omega t \qquad \varepsilon_0 = i_0\, R$$





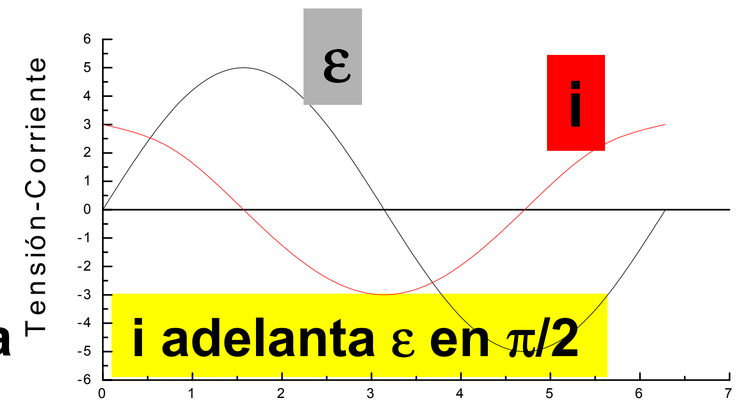
$$q = C\,V = C\,\varepsilon_0\ \text{sen}\ \omega t$$

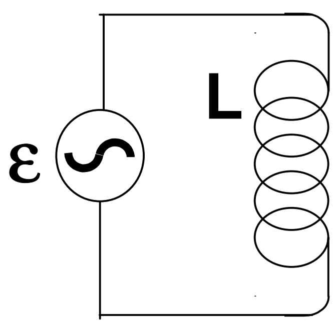
$$i = \frac{dq}{dt} = C\,\varepsilon_0\,\omega \cos \omega t$$

$$i = i_0\ \text{sen}\ (\omega t + \pi/2)$$

$$\varepsilon_0 = i_0\,(1/\omega C) \qquad X_C = \frac{1}{\omega C}$$

Reactancia capacitiva





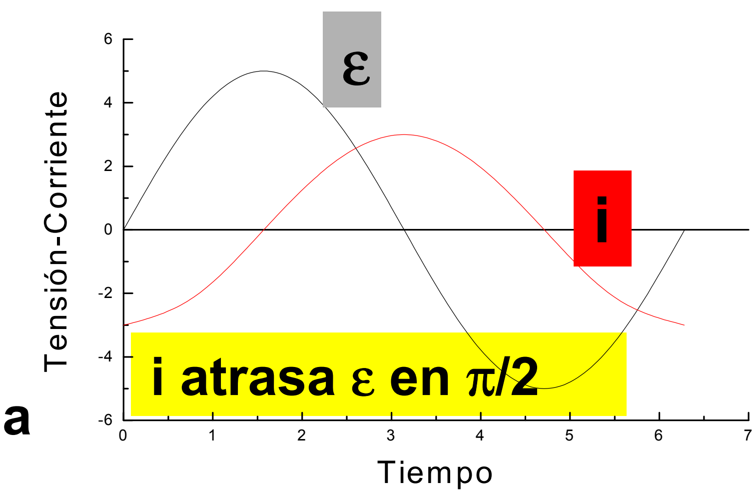
$$\varepsilon = L\frac{di}{dt} \Rightarrow i = \int \frac{\varepsilon}{L}\,dt$$

$$i = -\frac{\varepsilon_0}{\omega L}\cos \omega t$$

$$i = i_0\ \text{sen}\ (\omega t - \pi/2)$$

$$\varepsilon_0 = i_0\,\omega L \qquad X_L = \omega L$$

Reactancia inductiva



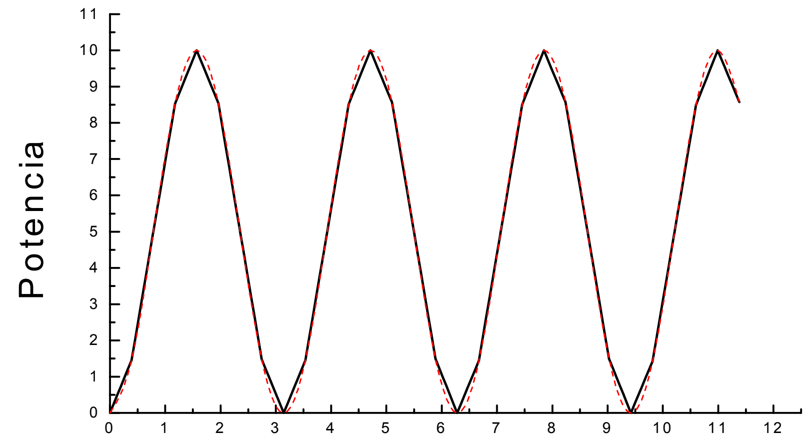
R

$$\varepsilon = \varepsilon_0 \operatorname{sen} \omega t$$

$$i = \frac{\varepsilon_0}{R} sen\, \omega t$$

$$P = i\, \varepsilon = \frac{\varepsilon_0^{\ 2}}{R} sen^2 \omega t$$

$$P_R = i^2 R = \frac{\varepsilon_0^{\ 2}}{R} sen^2 \omega t$$



C

$$i = \frac{dq}{dt} = C\, \varepsilon_0\, \omega \cos \omega t$$

$$q = C\, V = C\, \varepsilon_0 \operatorname{sen} \omega t$$

$$P = i\, \varepsilon = \varepsilon_0^{\ 2}\, C\, \omega\, sen\, \omega t \cos \omega t$$

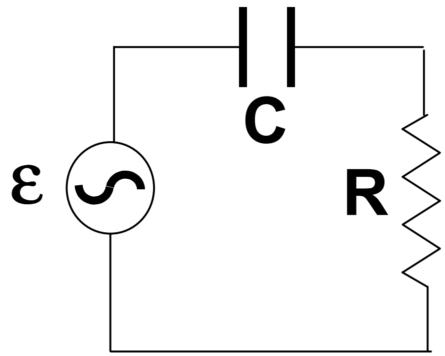
$$U_C = \frac{q^2}{2\, C} = \frac{C\, \varepsilon_0^{\ 2}}{2} sen^2 \omega t \qquad P_C = \frac{dU_C}{dt} = \varepsilon_0^{\ 2}\, C\, \omega\, sen\, \omega t \cos \omega t$$

L

$$i = -\frac{\varepsilon_0}{\omega L} \cos \omega t \qquad P = i\, \varepsilon = -\frac{\varepsilon_0^{\ 2}}{\omega L} sen \omega t \cos \omega t$$

$$U_L = \frac{L\, i^2}{2} = \frac{L\, \varepsilon_0^{\ 2}}{2\, (\omega L)^2} \cos^2 \omega t \qquad P_L = \frac{dU_L}{dt} = --\frac{\varepsilon_0^{\ 2}}{\omega L} sen\, \omega t \cos \omega t$$

Circuito RC alimentado con ca



$$\varepsilon = \varepsilon_0 \operatorname{sen} \omega t$$

$$i = i_0 \operatorname{sen} (\omega t + \varphi)$$

$$\varepsilon = i R + \frac{q}{C}$$

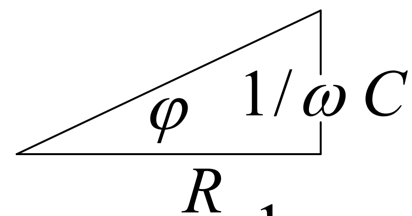
$$q = \int i \, dt = -\frac{i_0}{\omega} \cos (\omega t + \varphi)$$

$$\varepsilon_0 \, sen \, \omega t = i_0 R \, sen \, (\omega t + \varphi) - \frac{i_0}{\omega C} \cos (\omega t + \varphi)$$

$$\varepsilon_0 \, sen \, \omega t = i_0 R \, (sen \omega t \cos \varphi + \cos \omega t \, sen \varphi) - \frac{i_0}{\omega C} (\cos \omega t \cos \varphi - sen \omega t \, sen \varphi)$$

Comportamiento capa-citivo, i adelanta a ε

$$\cos \omega t \, (R \, i_0 \operatorname{sen} \varphi - \frac{i_0}{\omega C} \cos \varphi) = 0 \Rightarrow \operatorname{tg} \varphi = \frac{1/\omega C}{R}$$



$$\operatorname{sen} \omega t \, (-\varepsilon_0 + R \, i_0 \cos \varphi + \frac{i_0}{\omega C} \operatorname{sen} \varphi) = 0 \Rightarrow \varepsilon_0 = i_0 \, (R \cos \varphi + \frac{1}{\omega C} \operatorname{sen} \varphi)$$

$$\cos \varphi = R / \sqrt{R^2 + (1/\omega C)^2} \qquad sen \varphi = \left(\frac{1}{\omega C} \right) / \sqrt{R^2 + (1/\omega C)^2}$$

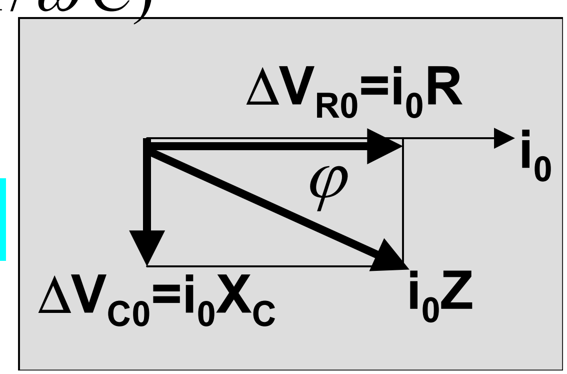
$$\varepsilon_0 = i_0 \sqrt{R^2 + (1/\omega C)^2} = i_0 \, Z$$

Z: Impedancia

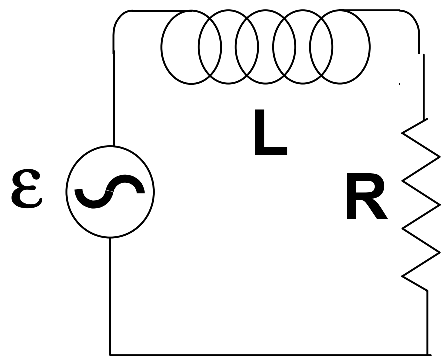
$$\Delta V_R = i \, R = (\varepsilon_0 \, R / Z) \operatorname{sen} (\omega t + \varphi)$$

$$\Delta V_R + \Delta V_C = \varepsilon$$

$$\Delta V_C = q / C = -(\varepsilon_0 / Z \, \omega C) \cos (\omega t + \varphi)$$



Circuito RL alimentado con ca



$$\varepsilon = \varepsilon_0 \text{ sen } \omega t$$

$$i = i_0 \text{ sen } (\omega t + \varphi)$$

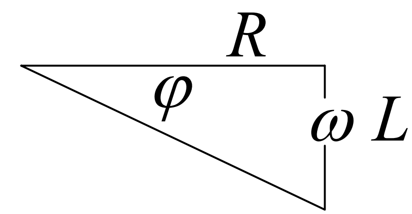
$$\varepsilon = i R + L \frac{di}{dt}$$

$$\varepsilon_0 \text{ sen } \omega t = i_0 R \text{ sen } (\omega t + \varphi) + L i_0 \omega \cos (\omega t + \varphi)$$

$$\varepsilon_0\, sen\, \omega t = i_0 R (sen\omega t \cos\varphi + \cos\omega t\, sen\varphi) + L \omega i_0 (\cos\omega t \cos\varphi - sen\omega t\, sen\varphi)$$

Comportamiento inductivo, ε adelanta a i

$$\cos\omega t (R i_0 \text{ sen } \varphi + L \omega i_0 \cos\varphi) = 0 \Rightarrow \text{tg}\,\varphi = -\frac{\omega L}{R}$$



$$\text{sen}\,\omega t (-\varepsilon_0 + R i_0 \cos\varphi - L \omega i_0 \text{ sen } \varphi) = 0 \Rightarrow \varepsilon_0 = i_0 (R \cos\varphi - L\omega \text{sen}\,\varphi)$$

$$\varepsilon = i_0 \left(R \frac{R}{\sqrt{R^2 + (\omega L)^2}} - \omega L \frac{-\omega L}{\sqrt{R^2 + (\omega L)^2}}\right)$$

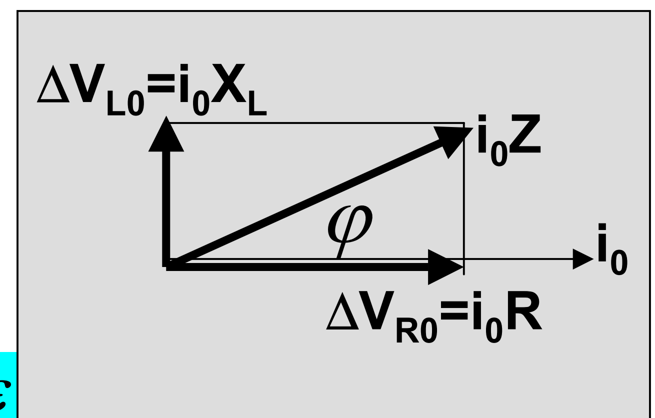
$$\varepsilon_0 = i_0 \sqrt{R^2 + (\omega L)^2} = i_0 Z$$

Z: Impedancia

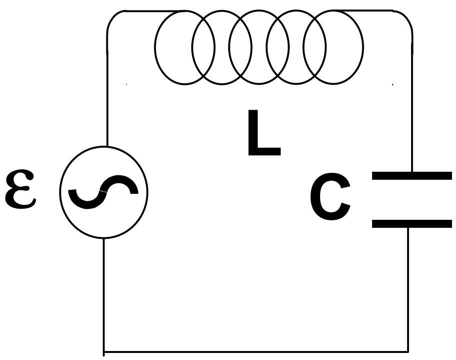
$$\Delta V_R = i R = (\varepsilon_0 R / Z) \text{ sen } (\omega t + \varphi)$$

$$\Delta V_L = L \frac{di}{dt} = \omega L \frac{\varepsilon_0}{Z} \cos (\omega t + \varphi)$$

$$\Delta V_R + \Delta V_L = \varepsilon$$



Circito LC alimentado con ca



$$\varepsilon = \varepsilon_0 \text{ sen } \omega t$$

$$i = i_0 \text{ sen } (\omega t + \varphi)$$

$$\varepsilon = \frac{q}{C} + L \frac{di}{dt}$$

$$q = \int_0^t i \, dt = -\frac{i_0}{\omega} (\cos(\omega t + \varphi) - \cos\varphi)$$

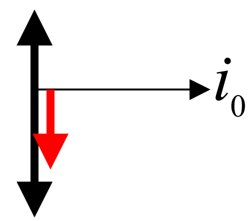
$$\varepsilon_0 \, sen \, \omega t = -(i_0 / \omega C)(\cos(\omega t + \varphi) - \cos\varphi) + L i_0 \omega \cos(\omega t + \varphi)$$

$$\varepsilon_0 \, sen \, \omega t = -(i_0 / \omega C)(\cos\omega t \cos\varphi - sen\omega t \, sen\varphi - \cos\varphi) + L \omega i_0 (\cos\omega t \cos\varphi - sen\omega t \, sen\varphi)$$

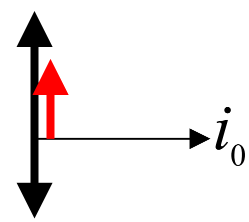
$$\varphi = \pm \frac{\pi}{2} \Rightarrow \cos\varphi = 0; sen\varphi = \pm 1$$

$$\text{sen}\, \omega t \, (-\varepsilon_0 + (i_0 / \omega C) \text{ sen} \, \varphi - L \omega i_0 \text{ sen} \, \varphi) = 0 \Rightarrow$$

$$\text{sen}\, \varphi = 1 \Rightarrow \varepsilon_0 = i_0 \left(\frac{1}{\omega C} - \omega L \right)$$ **Circuito capacitivo**

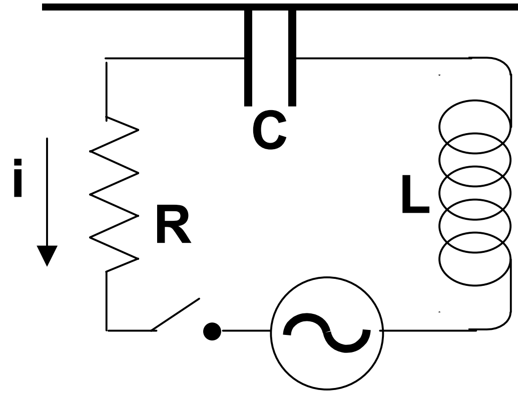


$$\text{sen}\, \varphi = -1 \Rightarrow \varepsilon_0 = i_0 \left(\omega L - \frac{1}{\omega C} \right)$$ **Circuito inductivo**



$$\varepsilon_0 = i_0 Z$$

Circuito RLC serie alimentado con ca



$$\varepsilon = \varepsilon_0 \text{ sen } \omega t$$

$$i = i_0 \; sen\,(\omega t + \varphi)$$

$$\varepsilon = i\,R + \frac{q}{C} + L\frac{di}{dt}$$

$$\varepsilon_0\omega \cos\omega t = L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i$$

$$\varepsilon_0\omega \cos\omega t = -L\,i_0\,\omega^2 sen(\omega t + \varphi) + R\,i_0\,\omega\cos(\omega t + \varphi) + \frac{i_0}{C}sen(\omega t + \varphi)$$

$$\varepsilon_0\omega \cos\omega t = -Li_0\omega^2(sen\,\omega t\cos\varphi + \cos\omega t\; sen\,\varphi) + Ri_0\omega$$

$$(\cos\omega t\cos\varphi - sen\,\omega t\; sen\,\varphi) + \frac{i_0}{C}(sen\,\omega t\cos\varphi + \cos\omega t\; sen\,\varphi)$$

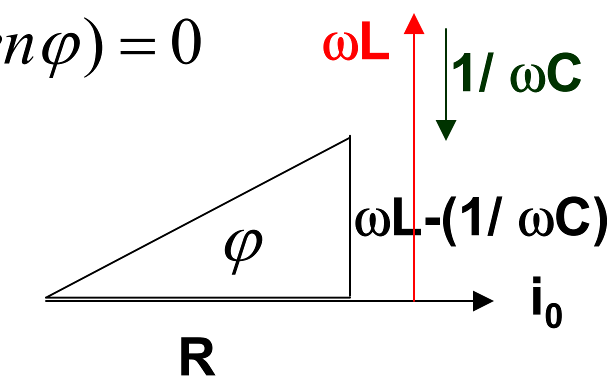
$$sen\omega t(-L\,i_0\,\omega^2\cos\varphi - R\,i_0\,\omega\, sen\varphi + \frac{i_0}{C}\cos\varphi) = 0$$

$$tg\varphi = \frac{\omega L - (1/\omega C)}{R}$$

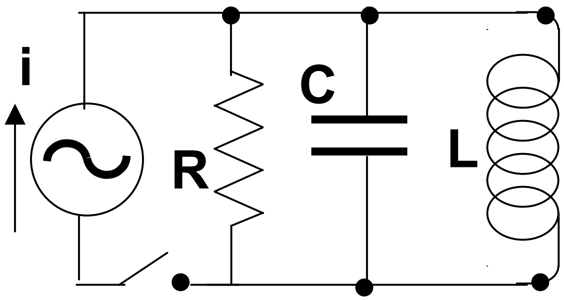
$$\cos\omega t\,(-\varepsilon_0\,\omega - L\,i_0\,\omega^2 sen\varphi + R\,i_0\,\omega\cos\varphi + \frac{i_0}{C}sen\varphi) = 0$$

$$\varepsilon_0 = i_0\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = i_0\,Z$$

$$i_0 = \varepsilon_0 / Z$$



Circuito RLC paralelo alimentado con ca



$$\varepsilon = \varepsilon_0 \operatorname{sen} \omega t$$

$$i = i_0 \, sen\,(\omega t + \varphi)$$

$$i = i_R + i_C + i_L$$

$$i_R = \frac{\varepsilon_0}{R} sen\,\omega t$$

$$i_C = \frac{\varepsilon_0}{(1/\omega C)} sen\,(\omega t + \frac{\pi}{2}) = \frac{\varepsilon_0}{(1/\omega C)} \cos \omega t$$

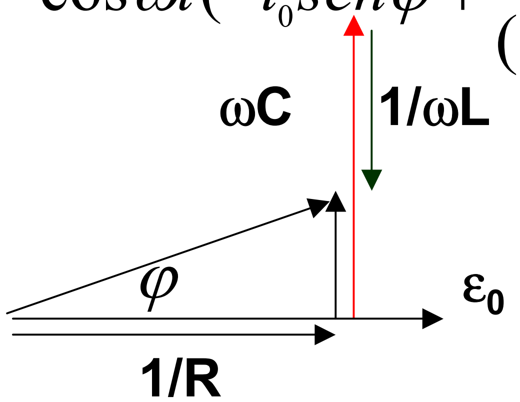
$$i_L = \frac{\varepsilon_0}{\omega L} sen\,(\omega t - \frac{\pi}{2}) = -\frac{\varepsilon_0}{\omega L} \cos \omega t$$

$$i_0 \, sen\,(\omega t + \varphi) = \frac{\varepsilon_0}{R} sen\,\omega t + \frac{\varepsilon_0}{(1/\omega C)} \cos \omega t - \frac{\varepsilon_0}{\omega L} \cos \omega t$$

$$tg\varphi = \frac{\omega C - 1/\omega L}{1/R}$$

$$sen\,\omega t(-i_0 \cos\varphi + \frac{\varepsilon_0}{R}) = 0 \qquad \cos\varphi = \frac{\varepsilon_0}{i_0 R}$$

$$\cos \omega t(-i_0 sen\,\varphi + \frac{\varepsilon_0}{(1/\omega C)} - \frac{\varepsilon_0}{\omega L}) = 0 \qquad sen\,\varphi = \frac{\varepsilon_0}{i_0}(\omega C - \frac{1}{\omega L})$$



$$\operatorname{sen}\varphi = \frac{\omega C - 1/\omega L}{\sqrt{(1/R)^2 + (\omega C - (1/\omega L))^2}}$$

$$\varepsilon_0 = i_0 \frac{1}{\sqrt{(1/R)^2 + (\omega C - (1/\omega L))^2}}$$

$$\varepsilon_0 = i_0 \, Z$$

Resonancia en circuito RLC

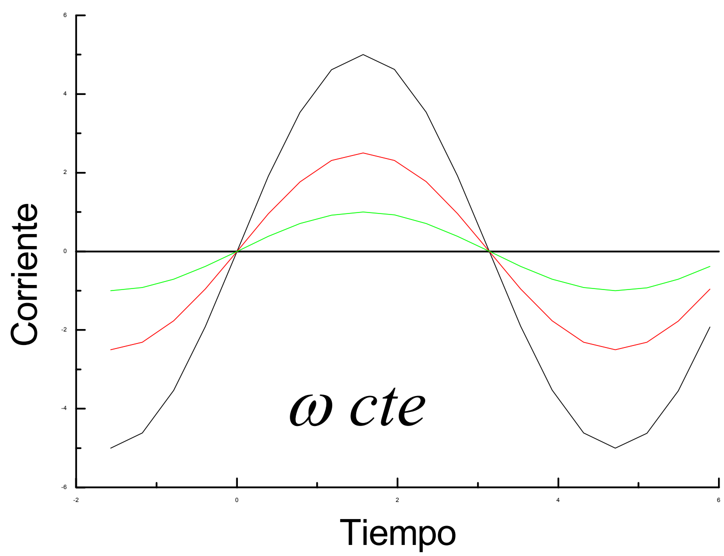
$$\varepsilon = \varepsilon_0 \operatorname{sen} \omega t \qquad i = i_0 \, sen\,(\omega t + \varphi)$$

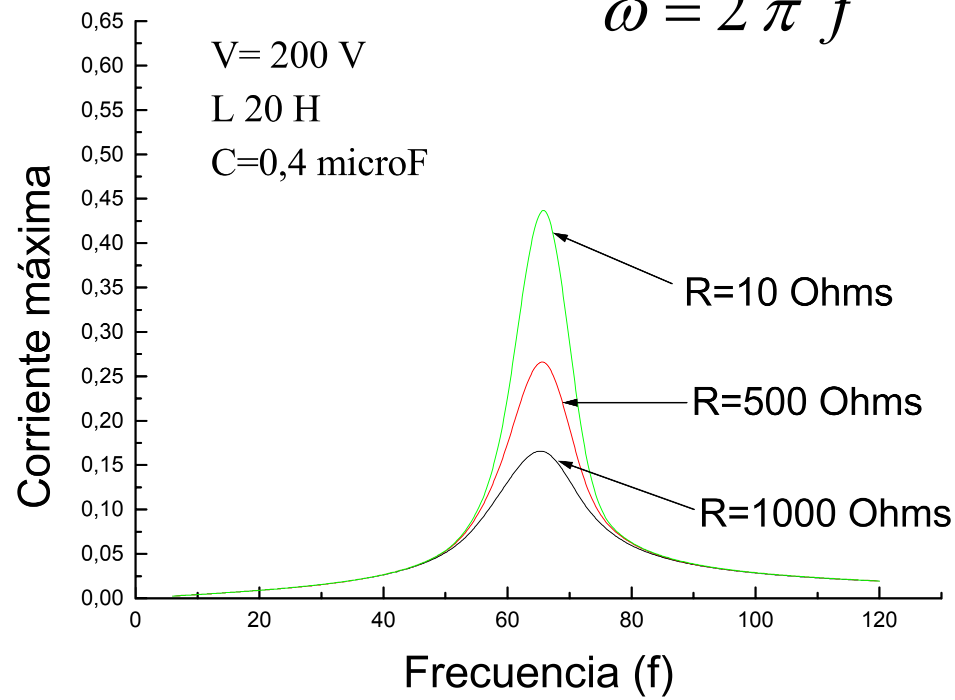
$$i_0 = \frac{\varepsilon_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$i_{máx} = \frac{\varepsilon_0}{R} \Rightarrow \omega L = \frac{1}{\omega C}$$

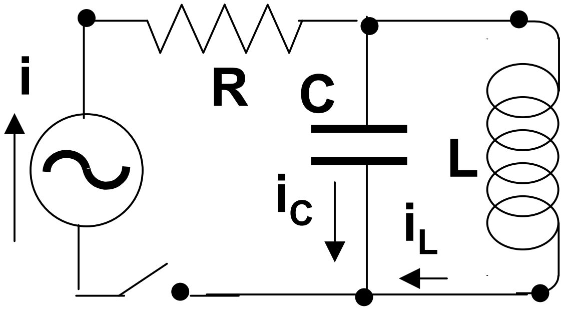
$$\omega = \frac{1}{\sqrt{L\,C}}$$

$$\omega = 2\pi f$$





Circuito tanque (CT)



RLC serie presenta algunas desventajas técnicas como sintonizador (allí resonancia es i max)

En CT sintonía con i mínimo. Como V_R será entonces mínimo, <u>la V sobre // LC es máxima</u>

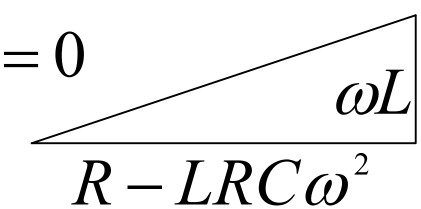
$$\varepsilon_0 sen\omega t = (i_L + i_C)R + L\frac{di_L}{dt} \qquad i_L = i_{0L} sen(\omega t + \varphi)$$

$$L\frac{di_L}{dt} = \frac{q_C}{C} \Rightarrow i_C = LC\frac{d^2 i_L}{dt^2} \qquad \varepsilon_0 sen\omega t = LRC\frac{d^2 i_L}{dt^2} + L\frac{di_L}{dt} + Ri_L$$

$$\varepsilon_0 sen\omega t = -LRCi_{0L}\omega^2(sen\omega t \cos\varphi + \cos\omega t \, sen\varphi) + \omega L i_{0L}(\cos\omega t \cos\varphi - sen\omega t \, sen\varphi) + Ri_{0L}(sen\omega t \cos\varphi + \cos\omega t \, sen\varphi)$$

$$\cos\omega t(-LRCi_{0L}\omega^2 sen\varphi + \omega L i_{0L}\cos\varphi + Ri_{0L} sen\varphi) = 0 \qquad \text{tg}\,\varphi = \frac{-\omega L}{R - LRC\omega^2}$$

$$sen\omega t\,(-\varepsilon_0 - LRCi_{0L}\omega^2\cos\varphi - \omega L i_{0L} sen\varphi + Ri_{0L}\cos\varphi) = 0$$



$$\varepsilon_0 = i_{0L}[(-LRC\omega^2 + R)\cos\varphi - L\omega \, sen\varphi]$$

$$\text{sen}\,\varphi = \frac{-\omega L}{[(R - LRC\omega^2)^2 + (\omega L)^2]^{1/2}} \qquad \cos\varphi = \frac{R - LRC\omega^2}{[(R - LRC\omega^2)^2 + (\omega L)^2]^{1/2}}$$

$$\varepsilon_0 = i_{0L}[(R - LRC\omega^2)^2 + (\omega L)^2]^{1/2} = i_{0L} Z_L \qquad i_{0L} = \frac{\varepsilon_0}{Z_L}$$

$$i_L = i_{0L} sen(\omega t + \varphi) \qquad i_C = LC\frac{d^2 i_L}{dt^2} = -LC i_{0L}\omega^2 \,\mathrm{sen}(\omega t + \varphi)$$

$$i = i_C + i_L = i_{0L}(1 - LC\omega^2)\,\mathrm{sen}(\omega t + \varphi) = i_0 \,\mathrm{sen}(\omega t + \varphi)$$

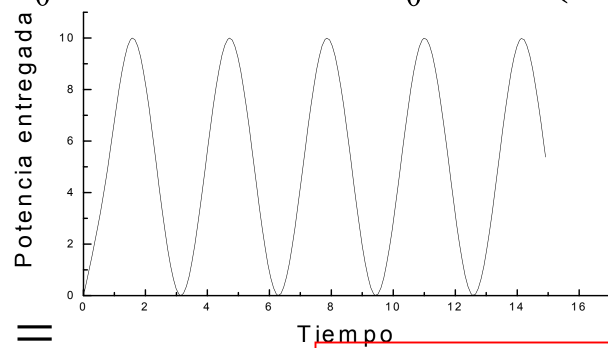
$$i_0 = \frac{\varepsilon_0 (1 - LC\omega^2)}{[(R - LRC\omega^2)^2 + (\omega L)^2]^{1/2}} = \frac{\varepsilon_0}{\sqrt{R^2 + \dfrac{L^2/C^2}{(\omega L - \dfrac{1}{\omega C})^2}}} = \frac{\varepsilon_0}{Z}$$

$$si\ \omega = \frac{1}{\sqrt{LC}} \Rightarrow i = 0$$

No hay caída en R y todo la tensión en //

Potencia en circuitos de alterna

$$\varepsilon = \varepsilon_0 \ \mathrm{sen}\ \omega t \qquad i = i_0 \ sen\,(\omega t + \varphi)$$



$$P(t) = i\,\varepsilon = i_0\ \varepsilon_0\ \mathrm{sen}\,\omega t\ \mathrm{sen}(\omega t + \varphi)$$

Sentido de potencia instantánea?

$$\overline{P} = \frac{1}{T}\int_0^T i\,\varepsilon\,dt = \frac{1}{T}\int_0^T i_0\ \varepsilon_0\ \mathrm{sen}\,\omega t\ \mathrm{sen}(\omega t + \varphi) =$$

$$\frac{i_0\ \varepsilon_0}{T}\left(\int_0^T \mathrm{sen}\,\omega t\ \mathrm{sen}\,\omega t\ \cos\varphi + \int_0^T \mathrm{sen}\,\omega t\ \cos\omega t\ \mathrm{sen}\,\varphi\right)$$

Función impar

$$\overline{P} = \frac{i_0\ \varepsilon_0}{2}\cos\varphi$$

$$i_{ef} = i_0 / \sqrt{2}$$

$$\varepsilon_{ef} = \varepsilon_0 / \sqrt{2}$$

Valores constantes de i y V que producen la misma potencia me-dia que la tensión de alterna

220 V de línea es Vef: 310 de pico

$\cos\varphi$ **Factor de potencia**

Al circuito inductivo o capacitivo (φ±π/2) no se le entrega potencia

$P_a = i_{ef}\ \varepsilon_{ef}\ \cos\varphi$ **Potencia activa (la que se disipa en R)** $\quad i_{ef}\varepsilon_{ef}\dfrac{R}{Z} = i_{ef}^{\ 2} R$

$P_r = i_{ef}\ \varepsilon_{ef}\ sen\varphi$ **Potencia reactiva (la que oscila)**

$P_{ap} = i_{ef}\ \varepsilon_{ef}$ **Potencia aparente**

Para una P requerida, a > cos φ < i.
Multas por bajo cos φ. Se cobre VA

Tratamiento con números complejos

Elementos en serie $V(t)=\sum V_k(t)$ **Elementos en paralelo** $i(t)=\sum i_k(t)$

Se definen tensión y corriente complejas $V_0 \neq \sum V_{k0}$ $i_0 \neq \sum i_{k0}$

$$\mathrm{E}=\varepsilon_0 e^{j\varphi_1}=\varepsilon_0\cos\varphi_1+j\varepsilon_0 sen\varphi_1=a+jb$$

$$|E|=\varepsilon_0=\sqrt{a^2+b^2} \qquad \varphi_1=arctg\frac{b}{a} \qquad V=\mathrm{Re}(Ee^{j\omega t})$$

$$I=i_0 e^{j\varphi_2}=i_0\cos\varphi_2+ji_0 sen\varphi_2=c+jd$$

$$|I|=i_0=\sqrt{c^2+d}^{\,2} \qquad \varphi_2=arctg(c/d) \qquad i=\mathrm{Re}(Ie^{j\omega t})$$

$$i_1+i_2=\mathrm{Re}[(I_1+I_2)e^{j\omega t}]$$

Si $\varepsilon=\varepsilon_0\cos\omega t$

En R $I=\frac{\varepsilon_0}{R}e^{jo}$ $\qquad I=\varepsilon_0/R$

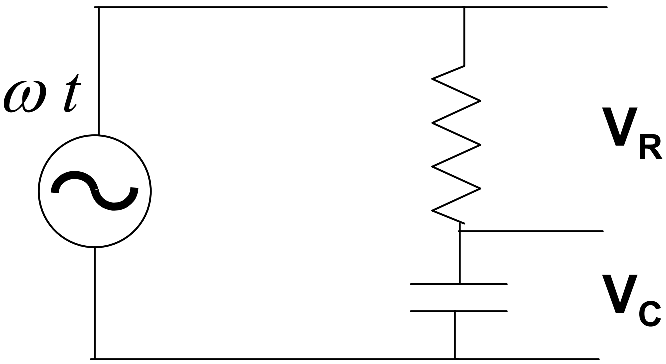
En C $I=\varepsilon_0\omega Ce^{j(\pi/2)}$ $\qquad I=j\varepsilon_0\omega C$

En L $I=\frac{\varepsilon_0}{\omega L}e^{-j(\pi/2)}$ $\qquad I=-j(\varepsilon_0/\omega L)$

	Z	**Y**	**Z***	**Y***
R	R	$1/R$	R	$1/R$
C	$1/\omega C$	ωC	$\frac{-j}{\omega C}$	$j\omega C$
L	ωL	$\frac{1}{\omega L}$	$j\omega L$	$\frac{-j}{\omega L}$

Pasa altos y pasa bajos

$$\varepsilon = \varepsilon_0 \operatorname{sen} \omega t$$



$$i = i_0 \operatorname{sen} (\omega t + \varphi) \qquad \varepsilon_0 = i_0 \sqrt{R^2 + (1/\omega C)^2}$$

$$\operatorname{tg} \varphi = \frac{1/\omega C}{R} \qquad q = \int i\, dt = -\frac{i_0}{\omega} \cos (\omega t + \varphi)$$

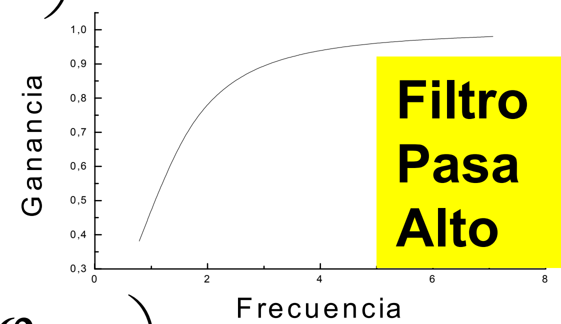
$$\Delta V_R = iR = R(\varepsilon_0 / Z) \operatorname{sen} (\omega t + \varphi) \qquad \Delta V_C = q/C = -(\varepsilon_0 / Z\, \omega\, C) \cos(\omega t + \varphi)$$

$$\frac{\overline{V}_R}{\varepsilon} = \frac{(\varepsilon_0\, R/Z)}{\tau\, \varepsilon_0} \left(\int_0^\tau \frac{\operatorname{sen} \omega t \cos \varphi}{\operatorname{sen} \omega t} dt + \int_0^\tau \operatorname{cot} g \omega t \operatorname{sen} \varphi\, dt \right)$$

$$\frac{\overline{V}_R}{\varepsilon} = \frac{R}{Z} \cos \varphi = \frac{R^2}{R^2 + (1/\omega C)^2}$$

$\longrightarrow$ $0 \quad si\ \omega \to 0$

$\longrightarrow$ $1 \quad si\ \omega \to \infty$



$$\frac{\overline{V}_C}{\varepsilon} = \frac{-(\varepsilon_0 / Z\, \omega C)}{\tau\, \varepsilon_0} \left(\int_0^\tau \frac{\cos \omega t \cos \varphi}{\operatorname{sen} \omega t} dt - \int_0^\tau \frac{\operatorname{sen} \omega t \operatorname{sen} \varphi}{\operatorname{sen} \omega t} dt \right)$$

$$\frac{\overline{V}_C}{\varepsilon} = \frac{1/\omega C}{Z} \operatorname{sen} \varphi = \frac{(1/\omega C)^2}{R^2 + (1/\omega C)^2}$$

$\longrightarrow$ $1 \quad si\ \omega \to 0$

$\longrightarrow$ $0 \quad si\ \omega \to \infty$