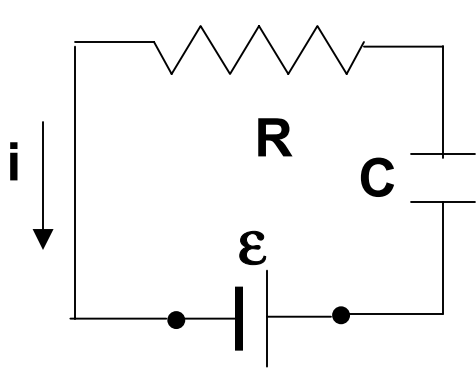


# Circuito RC alimentado con cc



$$\varepsilon = i R + \frac{q}{C} = R \frac{dq}{dt} + \frac{q}{C} \quad q = 0 \text{ en } t = 0 \Rightarrow i_0 = \frac{\varepsilon}{R}$$

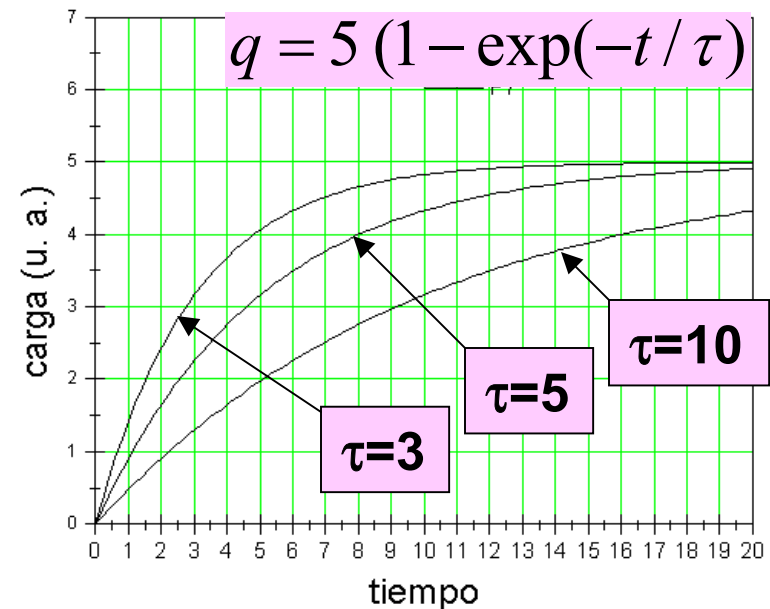
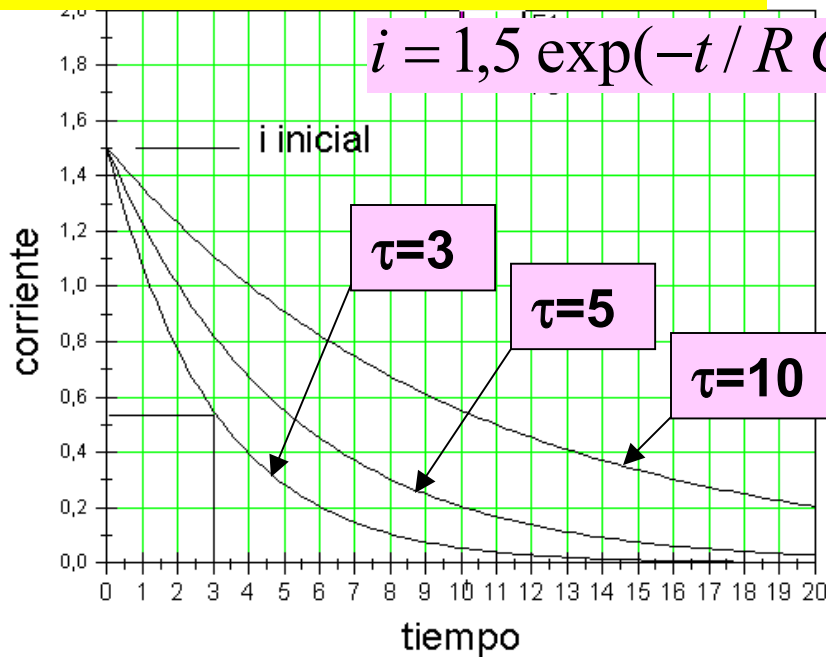
$$R \frac{di}{dt} + \frac{i}{C} = 0 \quad \frac{di}{i} = -\frac{1}{RC} dt$$

$$i = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$

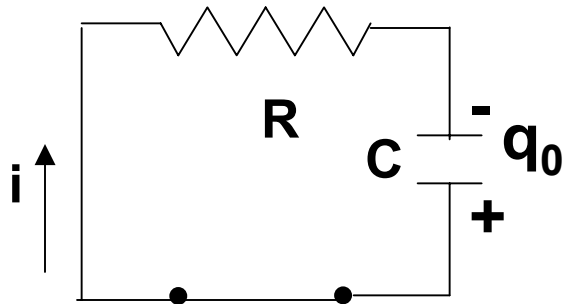
$$q = \int_0^t i dt = \frac{\varepsilon}{R} (-RC) e^{-\frac{t}{RC}} \Big|_0^t$$

$$q = \varepsilon C \left( 1 - e^{-\frac{t}{RC}} \right)$$

## $\tau = RC$ : constante de tiempo RC



## Curva de descarga del capacitor

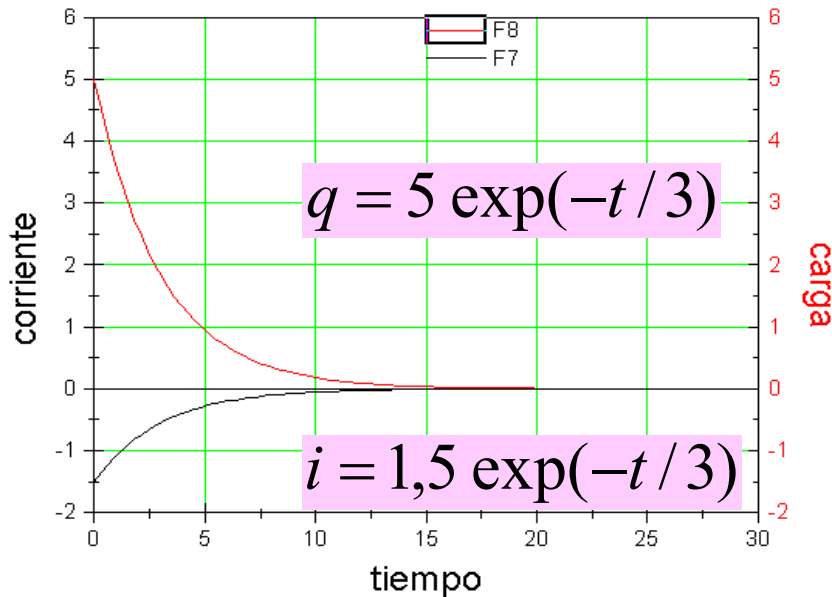


En determinado momento de la curva de carga se cortocircuita la fem, por ej. cuando  $q=q_0$

$$\frac{q}{C} + i' R = \frac{q}{C} + \frac{dq}{dt} R = 0$$

$$\frac{dq}{q} = -\frac{dt}{RC}$$

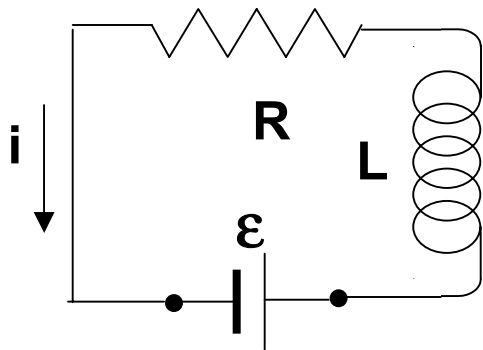
$$q = q_0 e^{-\frac{t}{RC}}$$



$$i = \frac{dq}{dt} = -\frac{q_0}{RC} e^{-\frac{t}{RC}}$$

**i en sentido contrario**

# Circuito RL alimentado con cc



$$\varepsilon = iR + L \frac{di}{dt}$$

$$R s + L \frac{ds}{dt} = 0$$

$$R \frac{di}{dt} + L \frac{d^2i}{dt^2} = 0 \quad s = \frac{di}{dt}$$

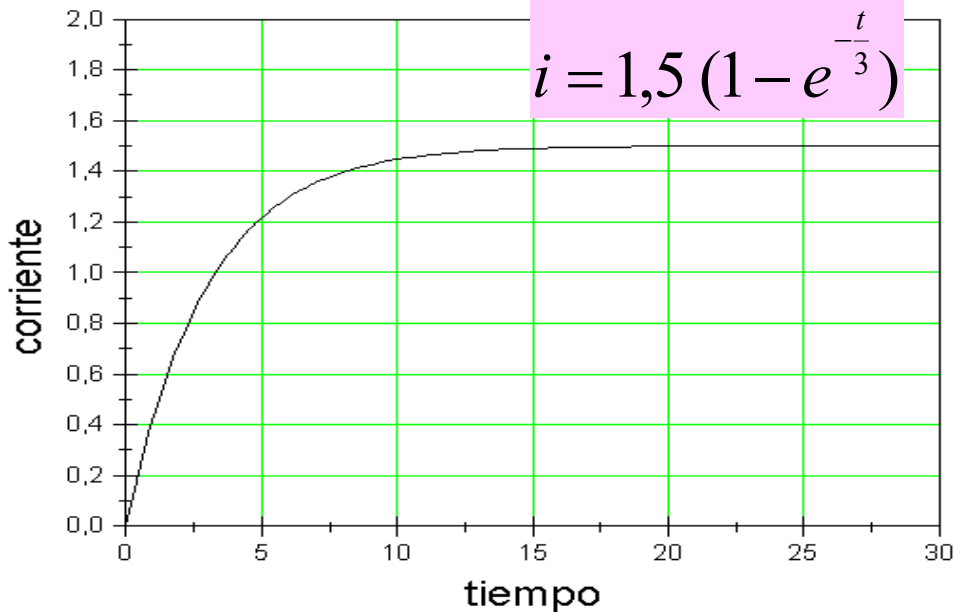
$$\frac{ds}{s} = -\frac{R}{L} dt \quad \ln \frac{s}{s_{t=0}} = -\frac{R}{L} t$$

En  $t=0$ ,  $i=0$  y  $(di/dt)_{t=0} = s_{t=0} = \varepsilon / L$

$$s = \frac{\varepsilon}{L} e^{-\frac{Rt}{L}}$$

$$i = \frac{\varepsilon}{R} \left( 1 - e^{-\frac{Rt}{L}} \right)$$

$\tau = L/R$ : constante de tiempo RL



Si en  $t'$  se cc la fem

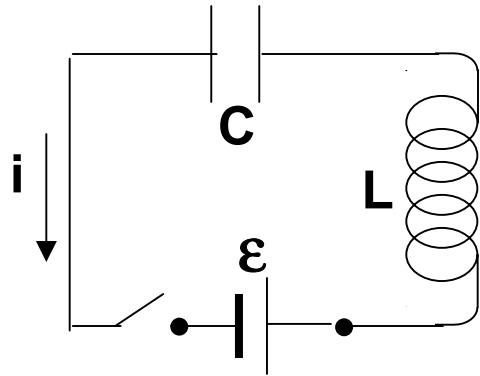
$$iR + L \frac{di}{dt} = 0 \quad i = i_0 e^{-\frac{Rt}{L}}$$

**Con CC**

**t = 0    C: cc    L: ca**

**t = ∞    C: ca    L: cc**

## Circuito LC alimentado con cc



$$\varepsilon - L \frac{di}{dt} - \frac{q}{C} = 0$$

$$i = i_0 \cos(\omega t + \varphi)$$

$$t = 0 \Rightarrow q = 0 \text{ y } i = 0$$

$$L \frac{d^2 i}{dt^2} + \frac{1}{C} i = 0$$

Oscilador armónico simple

$$\omega = \sqrt{\frac{1}{LC}} = 2\pi f = \frac{2\pi}{\tau}$$

$i_0$ : amplitud;  $f$ : frecuencia natural circ. LC;  $\varphi$ : ángulo de fase;  $\tau$ : período

$$t = 0 \quad i = 0 \Rightarrow i = i_0 \cos \varphi = 0 \Rightarrow \varphi = \pm \frac{\pi}{2}$$

En  $t=0$ , C es un cc y solo actúa L  $\Rightarrow i(t=0)=0$

$$t = 0 \quad q = 0 \Rightarrow \varepsilon = L \left( \frac{di}{dt} \right)_{t=0} = -i_0 \omega L \operatorname{sen} \varphi \Rightarrow i_0 = -\frac{\varepsilon}{\omega L} \operatorname{sen}^{-1} \varphi$$

En  $t=0$  todo  $\varepsilon$  en L; C descargado

$$\text{si } \varphi = \frac{\pi}{2} \Rightarrow i_0 < 0 \Rightarrow \varphi = -\frac{\pi}{2}$$

$$i = \frac{\varepsilon}{\omega L} \cos \left( \omega t - \frac{\pi}{2} \right) = \frac{\varepsilon}{\omega L} \operatorname{sen} \omega t$$

$$i_0 = \frac{\varepsilon}{\omega L} = \varepsilon \sqrt{\frac{C}{L}}$$

## Carga en el condensador

$$q = \int_0^t i dt = \int_0^t \frac{\varepsilon}{\omega L} \text{sen } \omega t = \frac{\varepsilon}{\omega^2 L} (1 - \cos \omega t) = \varepsilon C (1 - \cos \omega t)$$

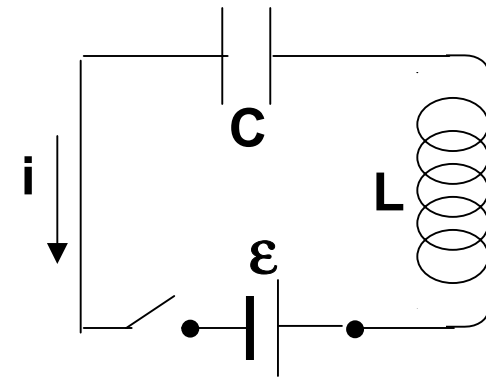
$$i = \frac{\varepsilon}{\omega L} \text{sen } \omega t$$

## Caída de potencial en el condensador

$$\Delta V_C = \frac{q}{C} = \varepsilon (1 - \cos \omega t)$$

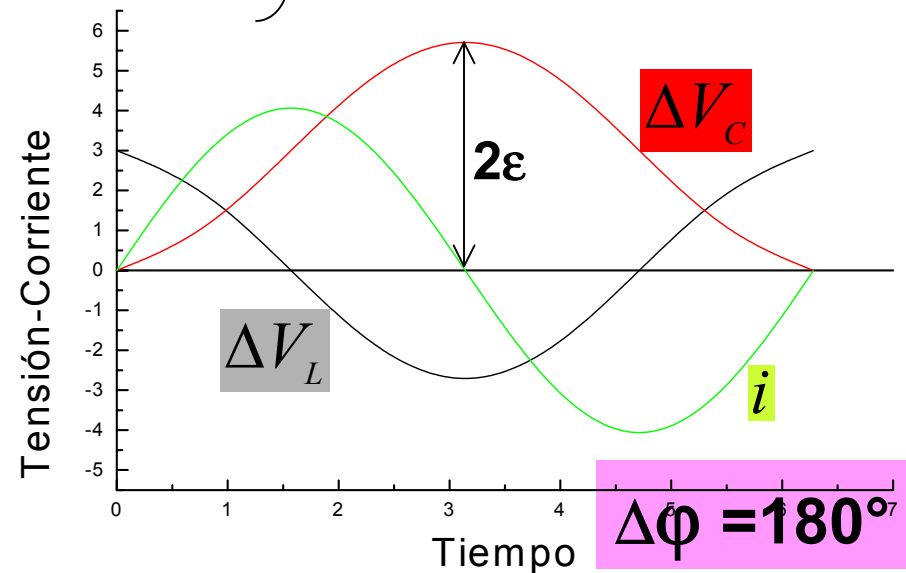
## Caída de potencial en el inductor

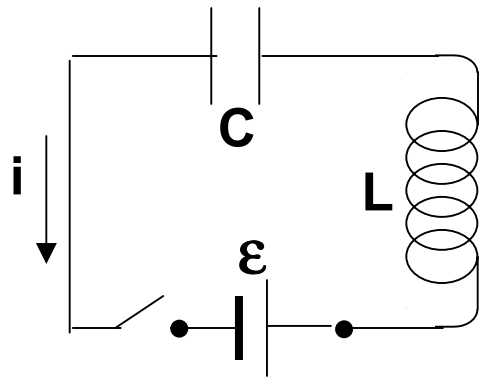
$$\Delta V_L = L \frac{di}{dt} = \varepsilon \cos \omega t$$



$$\Delta V_C + \Delta V_L = \varepsilon$$

Suma de tensiones parciales instantáneas en cada elemento =  $\varepsilon$ , pero no de amplitudes pues no tienen la misma fase





$$i = \frac{\varepsilon}{\omega L} \text{sen } \omega t$$

$$\Delta V_C = \frac{q}{C} = \varepsilon (1 - \cos \omega t)$$

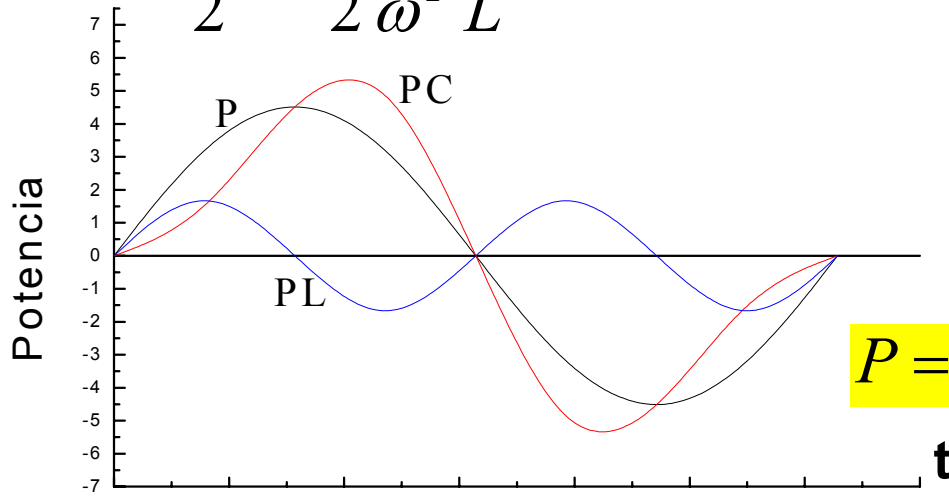
$$\Delta V_L = L \frac{di}{dt} = \varepsilon \cos \omega t$$

$$q = \varepsilon C (1 - \cos \omega t)$$

**Potencia suministrada**  $P = i \varepsilon = \frac{\varepsilon^2}{\omega L} \text{sen } \omega t = \varepsilon^2 \sqrt{\frac{C}{L}} \text{sen } \omega t$

$$U_C = \frac{q^2}{2C} = \frac{\varepsilon^2 C}{2} (1 - \cos \omega t)^2 \quad P_C = \frac{dU_C}{dt} = \frac{\varepsilon^2 C}{2} 2 (1 - \cos \omega t) \omega \text{sen } \omega t$$

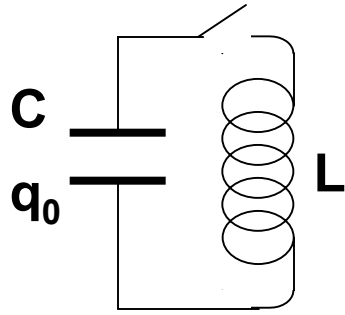
$$U_L = \frac{L i^2}{2} = \frac{\varepsilon^2}{2 \omega^2 L} \text{sen}^2 \omega t \quad P_L = \frac{dU_L}{dt} = \frac{\varepsilon^2}{2 \omega L} 2 \omega \text{sen } \omega t \cos \omega t$$



$$P_C = \varepsilon^2 \sqrt{\frac{C}{L}} (1 - \cos \omega t) \text{sen } \omega t$$

$$P_L = \varepsilon^2 \sqrt{\frac{C}{L}} \text{sen } \omega t \cos \omega t$$

$$P = P_C + P_L$$



$$L \frac{di}{dt} + \frac{q}{C} = 0 \Rightarrow L \frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

$$q = q_0 \cos(\omega t + \varphi)$$

$$\omega = 1/\sqrt{LC}$$

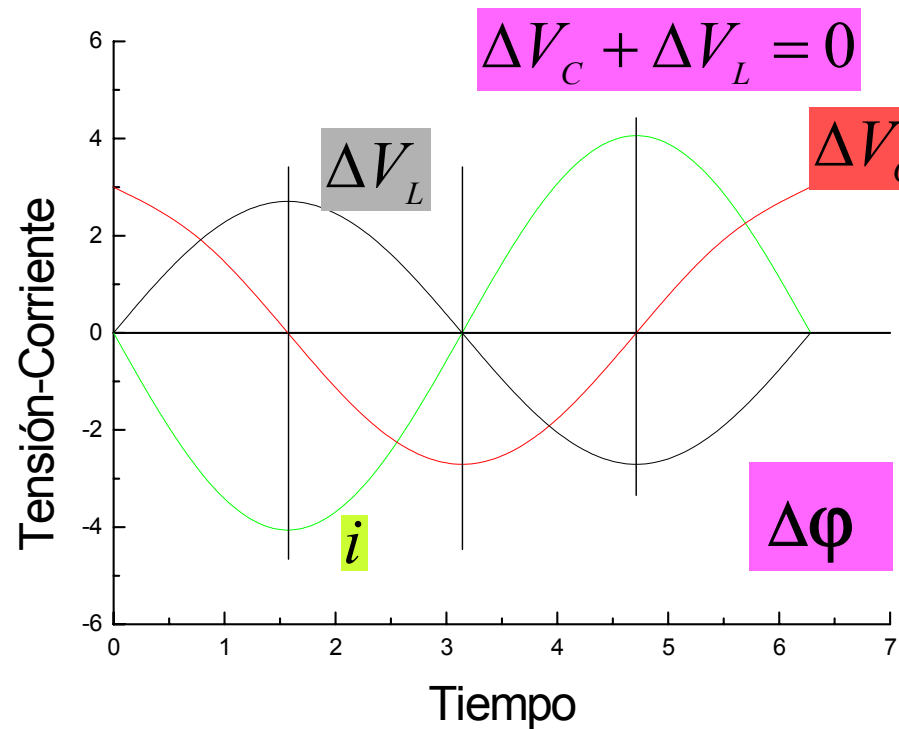
$$\text{Si } t = 0, q = q_0 \Rightarrow \varphi = 0$$

$$q = q_0 \cos \omega t$$

$$\Delta V_L = L \frac{di}{dt} = -\frac{q_0}{C} \cos \omega t = \frac{q_0}{C} \cos(\omega t - \pi)$$

$$i = \frac{dq}{dt} = -\omega q_0 \sin \omega t$$

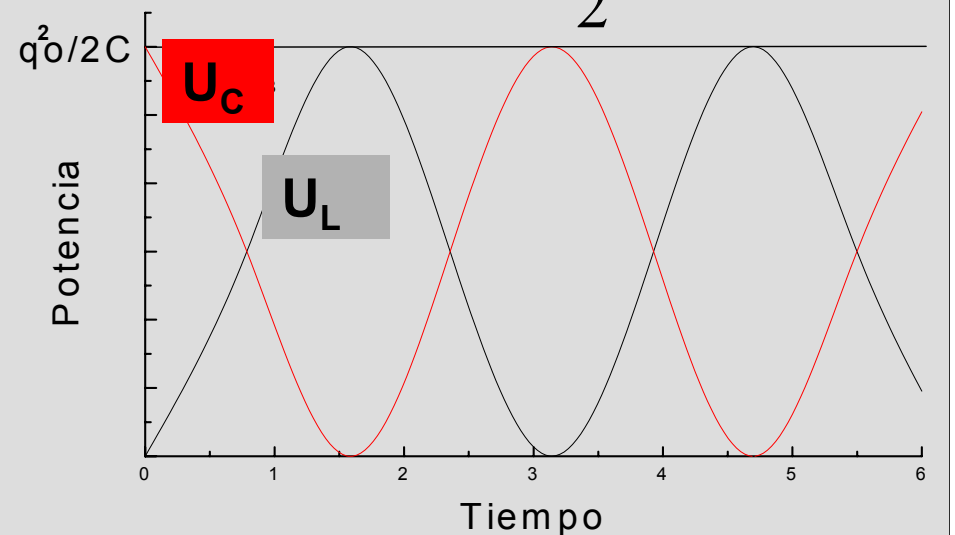
$$\Delta V_C = \frac{q}{C} = \frac{q_0}{C} \cos \omega t$$



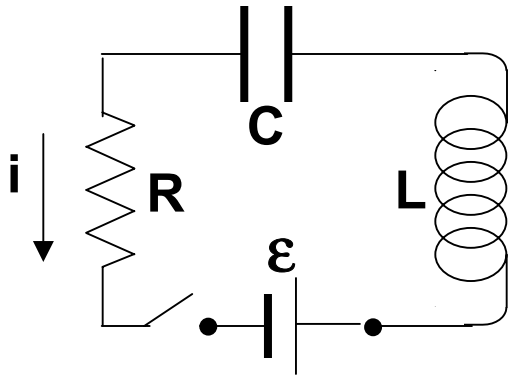
$$U = U_C + U_L = \frac{q_0^2}{2C} = \frac{q^2}{2C} + \frac{L i^2}{2}$$

$$U_C = \frac{q_0^2}{2C} \cos^2 \omega t$$

$$U_L = \frac{L \omega^2 q_0^2}{2} \sin^2 \omega t$$



# Circuito RLC alimentado por cc



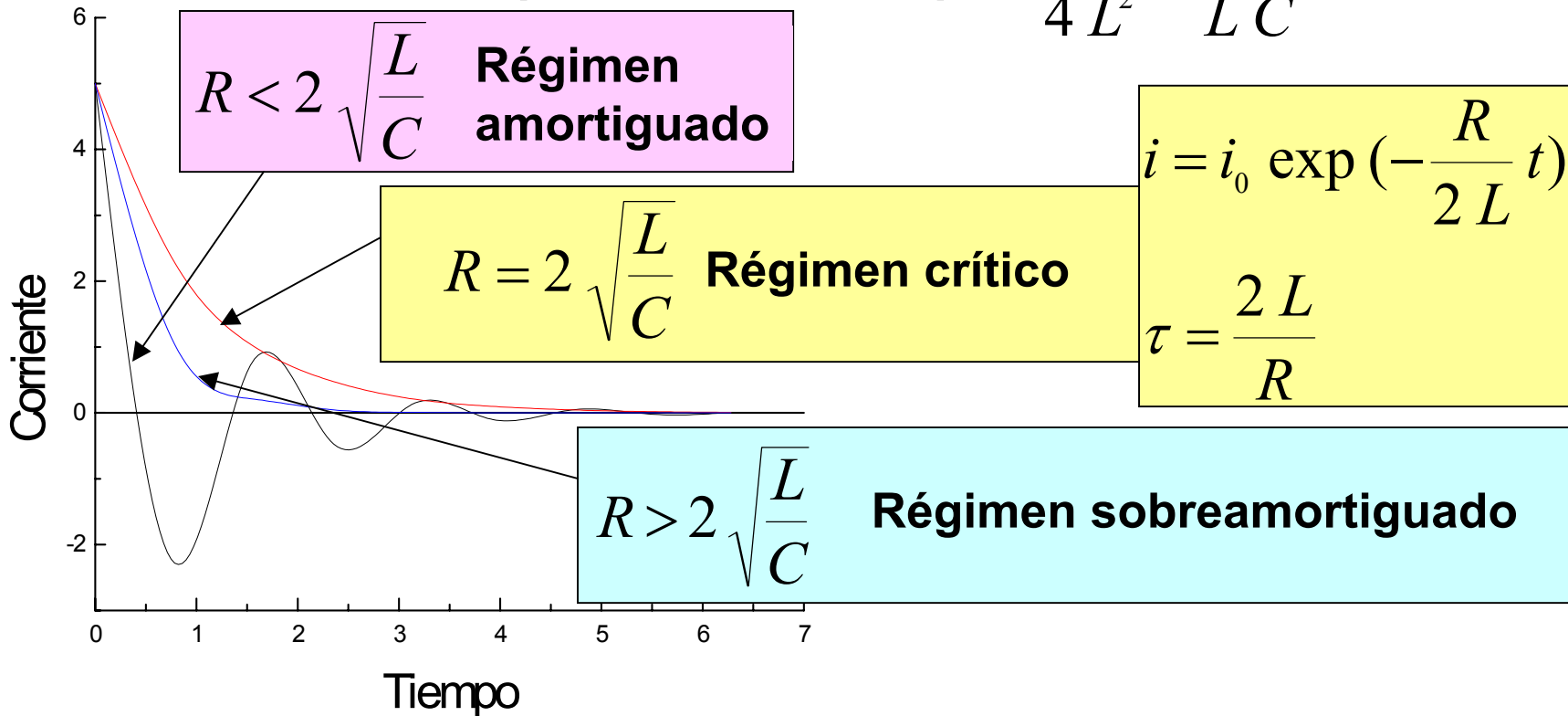
$$\varepsilon = L \frac{di}{dt} + R i + \frac{q}{C}$$

$$0 = L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{i}{C}$$

$$i = i_0 \exp\left(-\frac{R}{2L}t\right) \cos\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}t + \varphi\right)$$

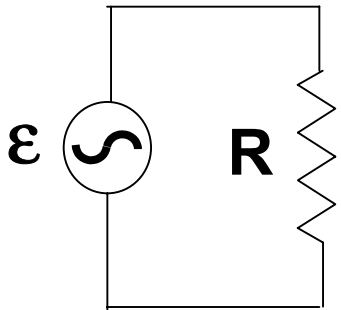
Solución solo válida para R chicas tal que

$$\frac{R^2}{4L^2} < \frac{1}{LC}$$



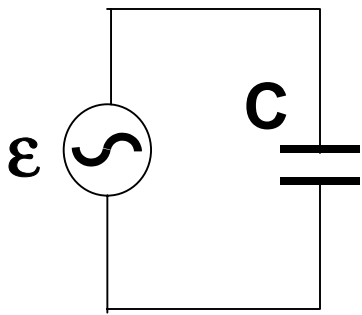
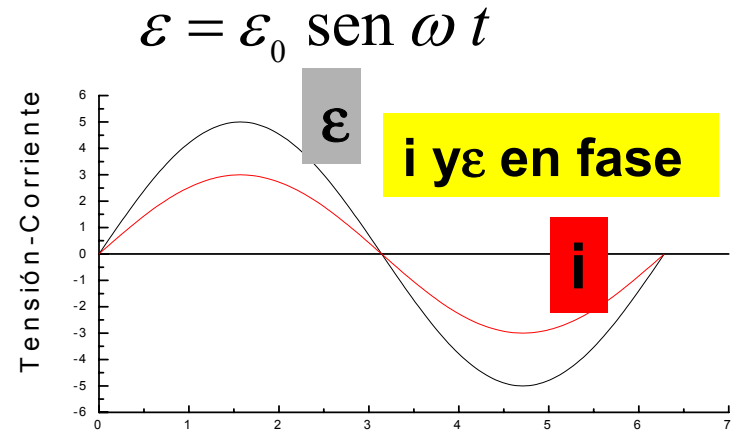


# R, C y L alimentadas con ca



$$i = \frac{\varepsilon}{R} = \frac{\varepsilon_0}{R} \text{ sen } \omega t$$

$$i = i_0 \text{ sen } \omega t \quad \varepsilon_0 = i_0 R$$



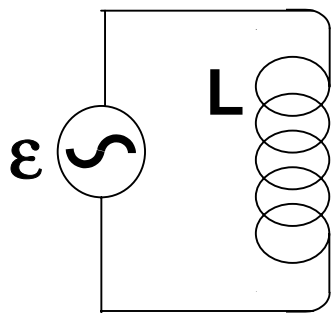
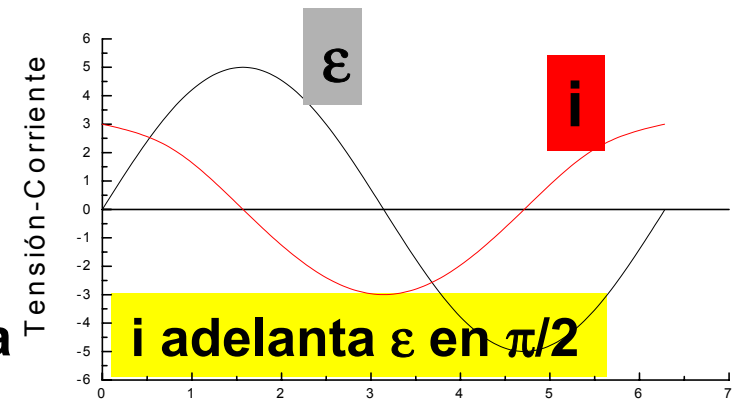
$$q = C V = C \varepsilon_0 \text{ sen } \omega t$$

$$i = \frac{dq}{dt} = C \varepsilon_0 \omega \text{ cos } \omega t$$

$$i = i_0 \text{ sen } (\omega t + \pi/2)$$

$$\varepsilon_0 = i_0 (1/\omega C)$$

$$X_C = \frac{1}{\omega C} \quad \text{Reactancia capacitiva}$$



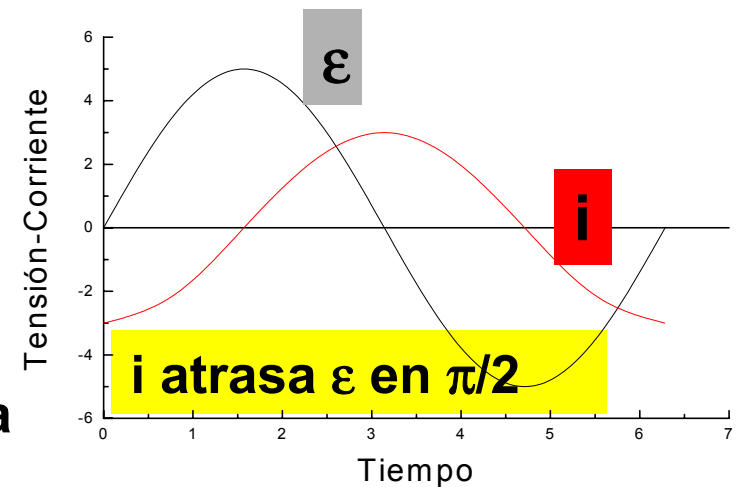
$$\varepsilon = L \frac{di}{dt} \Rightarrow i = \int \frac{\varepsilon}{L} dt$$

$$i = -\frac{\varepsilon_0}{\omega L} \text{ cos } \omega t$$

$$i = i_0 \text{ sen } (\omega t - \pi/2)$$

$$\varepsilon_0 = i_0 \omega L$$

$$X_L = \omega L \quad \text{Reactancia inductiva}$$

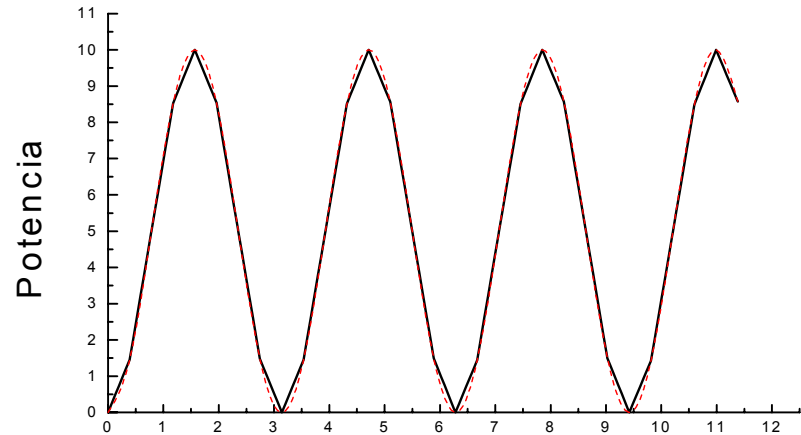


$$\mathbf{R} \quad \varepsilon = \varepsilon_0 \text{ sen } \omega t$$

$$i = \frac{\varepsilon_0}{R} \text{ sen } \omega t$$

$$P = i \varepsilon = \frac{\varepsilon_0^2}{R} \text{ sen}^2 \omega t$$

$$P_R = i^2 R = \frac{\varepsilon_0^2}{R} \text{ sen}^2 \omega t$$



$$\mathbf{C} \quad i = \frac{dq}{dt} = C \varepsilon_0 \omega \text{ cos } \omega t$$

$$q = C V = C \varepsilon_0 \text{ sen } \omega t$$

$$P = i \varepsilon = \varepsilon_0^2 C \omega \text{ sen } \omega t \text{ cos } \omega t$$

$$U_C = \frac{q^2}{2C} = \frac{C \varepsilon_0^2}{2} \text{ sen}^2 \omega t$$

$$P_C = \frac{dU_C}{dt} = \varepsilon_0^2 C \omega \text{ sen } \omega t \text{ cos } \omega t$$

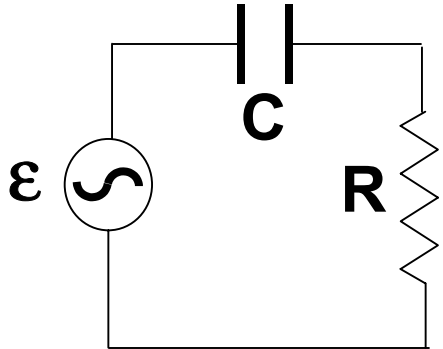
$$\mathbf{L} \quad i = -\frac{\varepsilon_0}{\omega L} \text{ cos } \omega t$$

$$P = i \varepsilon = -\frac{\varepsilon_0^2}{\omega L} \text{ sen } \omega t \text{ cos } \omega t$$

$$U_L = \frac{L i^2}{2} = \frac{L \varepsilon_0^2}{2 (\omega L)^2} \text{ cos}^2 \omega t$$

$$P_L = \frac{dU_L}{dt} = -\frac{\varepsilon_0^2}{\omega L} \text{ sen } \omega t \text{ cos } \omega t$$

## Circuito RC alimentado con ca



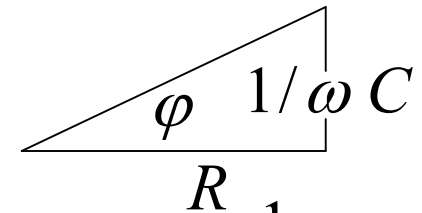
$$\varepsilon = i R + \frac{q}{C}$$

$$\varepsilon_0 \operatorname{sen} \omega t = i_0 R \operatorname{sen} (\omega t + \varphi) - \frac{i_0}{\omega C} \cos (\omega t + \varphi)$$

$$\varepsilon_0 \operatorname{sen} \omega t = i_0 R (\operatorname{sen} \omega t \cos \varphi + \cos \omega t \operatorname{sen} \varphi) - \frac{i_0}{\omega C} (\cos \omega t \cos \varphi - \operatorname{sen} \omega t \operatorname{sen} \varphi)$$

Comportamiento capacitivo,  $i$  adelanta a  $\varepsilon$

$$\cos \omega t \left( R i_0 \operatorname{sen} \varphi - \frac{i_0}{\omega C} \cos \varphi \right) = 0 \Rightarrow \operatorname{tg} \varphi = \frac{1/\omega C}{R}$$



$$\operatorname{sen} \omega t \left( -\varepsilon_0 + R i_0 \cos \varphi + \frac{i_0}{\omega C} \operatorname{sen} \varphi \right) = 0 \Rightarrow \varepsilon_0 = i_0 \left( R \cos \varphi + \frac{1}{\omega C} \operatorname{sen} \varphi \right)$$

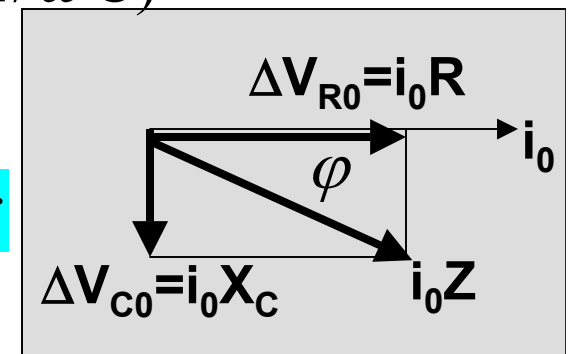
$$\cos \varphi = R / \sqrt{R^2 + (1/\omega C)^2} \quad \operatorname{sen} \varphi = \left( \frac{1}{\omega C} \right) / \sqrt{R^2 + (1/\omega C)^2}$$

$$\varepsilon_0 = i_0 \sqrt{R^2 + (1/\omega C)^2} = i_0 Z$$

**Z: Impedancia**

$$\Delta V_R = i R = (\varepsilon_0 R/Z) \operatorname{sen} (\omega t + \varphi) \quad \Delta V_R + \Delta V_C = \varepsilon$$

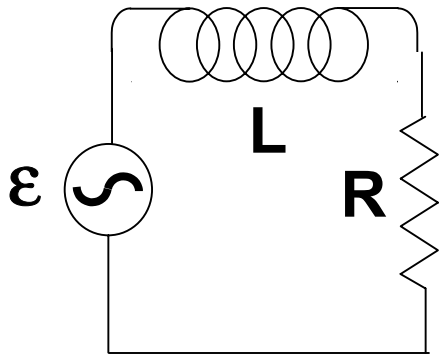
$$\Delta V_C = q/C = -(\varepsilon_0/Z \omega C) \cos (\omega t + \varphi)$$



## Circuito RL alimentado con ca

$$\varepsilon = \varepsilon_0 \text{ sen } \omega t$$

$$i = i_0 \text{ sen } (\omega t + \varphi)$$



$$\varepsilon = i R + L \frac{di}{dt}$$

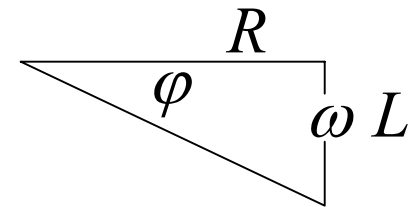
$$\varepsilon_0 \text{ sen } \omega t = i_0 R \text{ sen } (\omega t + \varphi) + L i_0 \omega \cos (\omega t + \varphi)$$

$$\varepsilon_0 \text{ sen } \omega t = i_0 R (\text{sen } \omega t \cos \varphi + \cos \omega t \text{ sen } \varphi) + L \omega i_0 (\cos \omega t \cos \varphi - \text{sen } \omega t \text{ sen } \varphi)$$

**Comportamiento inductivo,  $\varepsilon$  adelanta a  $i$**

$$\cos \omega t (R i_0 \text{ sen } \varphi + L \omega i_0 \cos \varphi) = 0 \Rightarrow$$

$$\text{tg } \varphi = -\frac{\omega L}{R}$$



$$\text{sen } \omega t (-\varepsilon_0 + R i_0 \cos \varphi - L \omega i_0 \text{ sen } \varphi) = 0 \Rightarrow \varepsilon_0 = i_0 (R \cos \varphi - L \omega \text{ sen } \varphi)$$

$$\varepsilon = i_0 \left( R \frac{R}{\sqrt{R^2 + (\omega L)^2}} - \omega L \frac{-\omega L}{\sqrt{R^2 + (\omega L)^2}} \right)$$

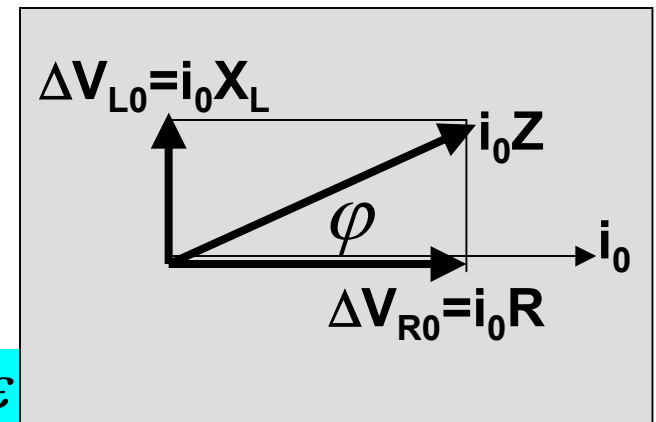
$$\varepsilon_0 = i_0 \sqrt{R^2 + (\omega L)^2} = i_0 Z$$

**Z: Impedancia**

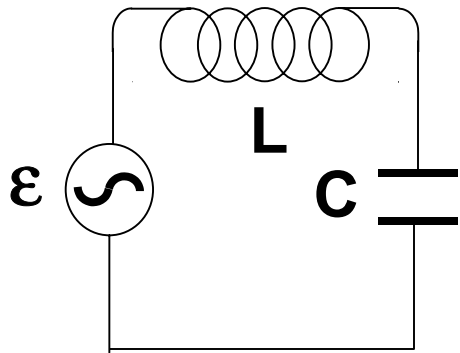
$$\Delta V_R = i R = (\varepsilon_0 R/Z) \text{ sen } (\omega t + \varphi)$$

$$\Delta V_L = L \frac{di}{dt} = \omega L \frac{\varepsilon_0}{Z} \cos (\omega t + \varphi)$$

$$\Delta V_R + \Delta V_L = \varepsilon$$



## Circuito LC alimentado con ca



$$\varepsilon = \frac{q}{C} + L \frac{di}{dt}$$

$$q = \int_0^t i dt = -\frac{i_0}{\omega} (\cos(\omega t + \varphi) - \cos \varphi)$$

$$\varepsilon_0 \text{ sen } \omega t = -(i_0 / \omega C) (\cos(\omega t + \varphi) - \cos \varphi) + L i_0 \omega \cos(\omega t + \varphi)$$

$$\varepsilon = \varepsilon_0 \text{ sen } \omega t$$

$$i = i_0 \text{ sen } (\omega t + \varphi)$$

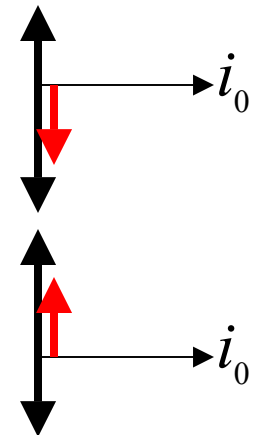
$$\varepsilon_0 \text{ sen } \omega t = -(i_0 / \omega C) (\cos \omega t \cos \varphi - \text{sen } \omega t \text{ sen } \varphi - \cos \varphi) + L \omega i_0 (\cos \omega t \cos \varphi - \text{sen } \omega t \text{ sen } \varphi)$$

$$\varphi = \pm \frac{\pi}{2} \Rightarrow \cos \varphi = 0; \text{ sen } \varphi = \pm 1$$

$$\text{sen } \omega t (-\varepsilon_0 + (i_0 / \omega C) \text{ sen } \varphi - L \omega i_0 \text{ sen } \varphi) = 0 \Rightarrow$$

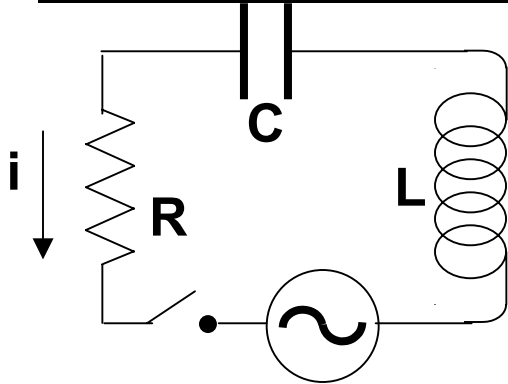
$$\text{sen } \varphi = 1 \Rightarrow \varepsilon_0 = i_0 \left( \frac{1}{\omega C} - \omega L \right) \quad \text{Circuito capacitivo}$$

$$\text{sen } \varphi = -1 \Rightarrow \varepsilon_0 = i_0 \left( \omega L - \frac{1}{\omega C} \right) \quad \text{Circuito inductivo}$$



$$\varepsilon_0 = i_0 Z$$

## Circuito RLC serie alimentado con ca



$$\varepsilon = i R + \frac{q}{C} + L \frac{di}{dt}$$

$$\varepsilon_0 \omega \cos \omega t = L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{1}{C} i$$

$$\varepsilon = \varepsilon_0 \text{ sen } \omega t$$

$$i = i_0 \text{ sen } (\omega t + \varphi)$$

$$\varepsilon_0 \omega \cos \omega t = -L i_0 \omega^2 \text{ sen}(\omega t + \varphi) + R i_0 \omega \cos(\omega t + \varphi) + \frac{i_0}{C} \text{ sen}(\omega t + \varphi)$$

$$\varepsilon_0 \omega \cos \omega t = -L i_0 \omega^2 (\text{sen } \omega t \cos \varphi + \cos \omega t \text{ sen } \varphi) + R i_0 \omega$$

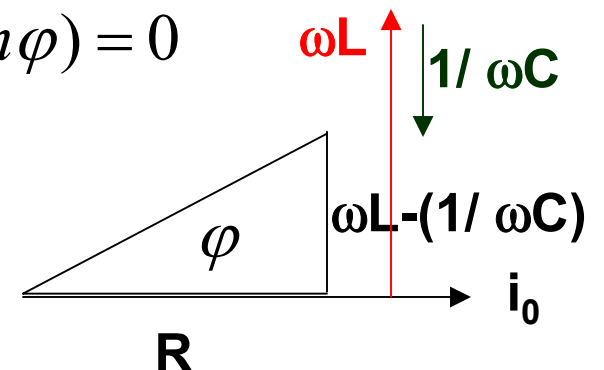
$$(\cos \omega t \cos \varphi - \text{sen } \omega t \text{ sen } \varphi) + \frac{i_0}{C} (\text{sen } \omega t \cos \varphi + \cos \omega t \text{ sen } \varphi)$$

$$\text{sen } \omega t (-L i_0 \omega^2 \cos \varphi - R i_0 \omega \text{ sen } \varphi + \frac{i_0}{C} \cos \varphi) = 0 \quad \text{tg } \varphi = \frac{\omega L - (1/\omega C)}{R}$$

$$\cos \omega t (-\varepsilon_0 \omega - L i_0 \omega^2 \text{ sen } \varphi + R i_0 \omega \cos \varphi + \frac{i_0}{C} \text{ sen } \varphi) = 0$$

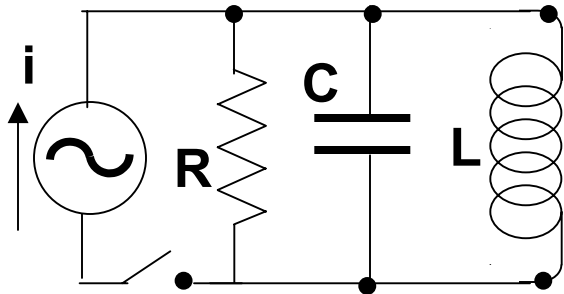
$$\varepsilon_0 = i_0 \sqrt{R^2 + \left( \omega L - \frac{1}{\omega C} \right)^2} = i_0 Z$$

$$i_0 = \varepsilon_0 / Z$$



## Circuito RLC paralelo alimentado con ca

$$\varepsilon = \varepsilon_0 \text{ sen } \omega t$$



$$i = i_0 \text{ sen } (\omega t + \varphi)$$

$$i = i_R + i_C + i_L$$

$$i_R = \frac{\varepsilon_0}{R} \text{ sen } \omega t$$

$$i_C = \frac{\varepsilon_0}{(1/\omega C)} \text{ sen } \left( \omega t + \frac{\pi}{2} \right) = \frac{\varepsilon_0}{(1/\omega C)} \text{ cos } \omega t$$

$$i_L = \frac{\varepsilon_0}{\omega L} \text{ sen } \left( \omega t - \frac{\pi}{2} \right) = -\frac{\varepsilon_0}{\omega L} \text{ cos } \omega t$$

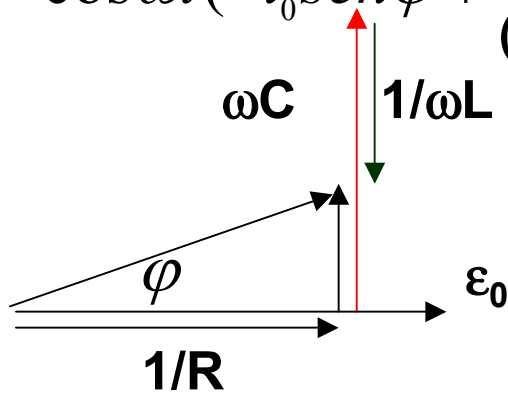
$$i_0 \text{ sen } (\omega t + \varphi) = \frac{\varepsilon_0}{R} \text{ sen } \omega t + \frac{\varepsilon_0}{(1/\omega C)} \text{ cos } \omega t - \frac{\varepsilon_0}{\omega L} \text{ cos } \omega t$$

$$\text{sen } \omega t \left( -i_0 \text{ cos } \varphi + \frac{\varepsilon_0}{R} \right) = 0$$

$$\text{cos } \varphi = \frac{\varepsilon_0}{i_0 R}$$

$$\text{tg } \varphi = \frac{\omega C - 1/\omega L}{1/R}$$

$$\text{cos } \omega t \left( -i_0 \text{ sen } \varphi + \frac{\varepsilon_0}{(1/\omega C)} - \frac{\varepsilon_0}{\omega L} \right) = 0 \quad \text{sen } \varphi = \frac{\varepsilon_0}{i_0} \left( \omega C - \frac{1}{\omega L} \right)$$



$$\text{sen } \varphi = \frac{\omega C - 1/\omega L}{\sqrt{(1/R)^2 + (\omega C - 1/\omega L)^2}}$$

$$\varepsilon_0 = i_0 \frac{1}{\sqrt{(1/R)^2 + (\omega C - 1/\omega L)^2}}$$

$$\varepsilon_0 = i_0 Z$$

# Resonancia en circuito RLC

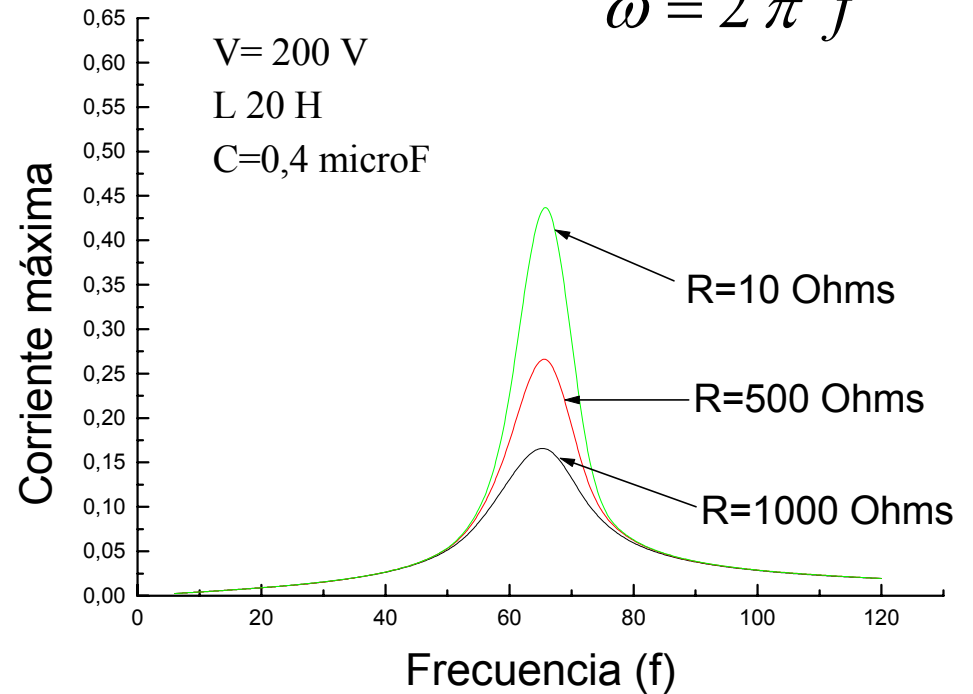
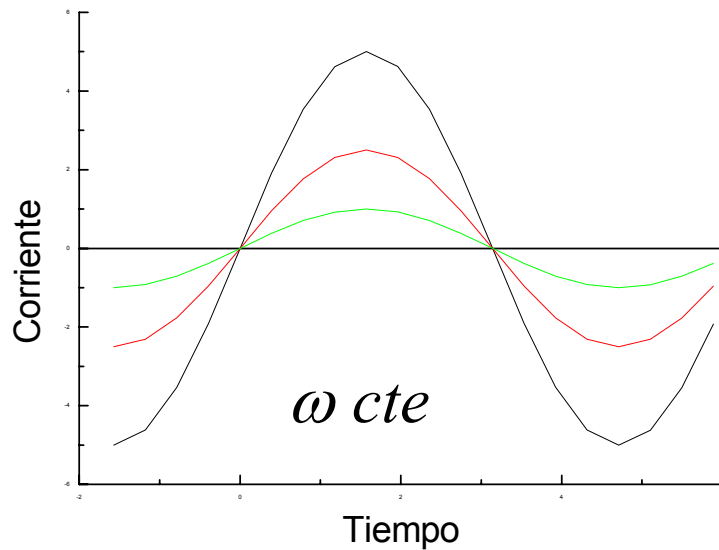
$$\varepsilon = \varepsilon_0 \text{ sen } \omega t \quad i = i_0 \text{ sen } (\omega t + \varphi)$$

$$i_0 = \frac{\varepsilon_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$i_{\text{m}\acute{a}\text{x}} = \frac{\varepsilon_0}{R} \Rightarrow \omega L = \frac{1}{\omega C}$$

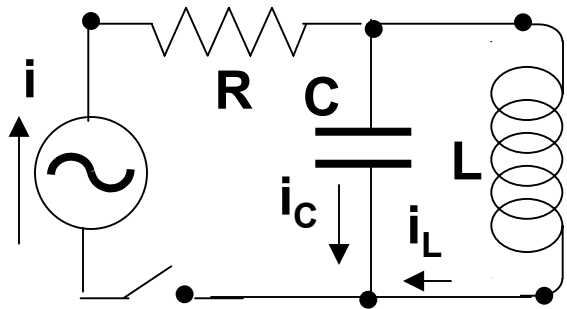
$$\omega = \frac{1}{\sqrt{LC}}$$

$$\omega = 2\pi f$$





## Circuito tanque (CT)



**RLC serie presenta algunas desventajas técnicas como sintonizador (allí resonancia es  $i$  max)**

**En CT sintonía con  $i$  mínimo. Como  $V_R$  será entonces mínimo, la  $V$  sobre // LC es máxima**

$$\varepsilon_0 \sin \omega t = (i_L + i_C)R + L \frac{di_L}{dt} \quad i_L = i_{0L} \sin(\omega t + \varphi)$$

$$L \frac{di_L}{dt} = \frac{q_C}{C} \Rightarrow i_C = LC \frac{d^2 i_L}{dt^2}$$

$$\varepsilon_0 \sin \omega t = LRC \frac{d^2 i_L}{dt^2} + L \frac{di_L}{dt} + Ri_L$$

$$\varepsilon_0 \sin \omega t = -LRCi_{0L} \omega^2 (\sin \omega t \cos \varphi + \cos \omega t \sin \varphi) + \omega Li_{0L} (\cos \omega t \cos \varphi - \sin \omega t \sin \varphi) + Ri_{0L} (\sin \omega t \cos \varphi + \cos \omega t \sin \varphi)$$

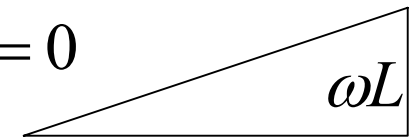
$$\cos \omega t (-LRCi_{0L} \omega^2 \sin \varphi + \omega Li_{0L} \cos \varphi + Ri_{0L} \sin \varphi) = 0 \quad \text{tg } \varphi = \frac{-\omega L}{R - LRC\omega^2}$$

$$\sin \omega t (-\varepsilon_0 - LRCi_{0L} \omega^2 \cos \varphi - \omega Li_{0L} \sin \varphi + Ri_{0L} \cos \varphi) = 0$$

$$\varepsilon_0 = i_{0L} [(-LRC\omega^2 + R) \cos \varphi - L\omega \sin \varphi]$$

$$\sin \varphi = \frac{-\omega L}{[(R - LRC\omega^2)^2 + (\omega L)^2]^{1/2}}$$

$$\cos \varphi = \frac{R - LRC\omega^2}{[(R - LRC\omega^2)^2 + (\omega L)^2]^{1/2}}$$



$$\varepsilon_0 = i_{0L} [(R - LRC\omega^2)^2 + (\omega L)^2]^{1/2} = i_{0L} Z_L \quad i_{0L} = \frac{\varepsilon_0}{Z_L}$$

$$i_L = i_{0L} \text{sen}(\omega t + \varphi) \quad i_C = LC \frac{d^2 i_L}{dt^2} = -LC i_{0L} \omega^2 \text{sen}(\omega t + \varphi)$$

$$i = i_C + i_L = i_{0L} (1 - LC\omega^2) \text{sen}(\omega t + \varphi) = i_0 \text{sen}(\omega t + \varphi)$$

$$i_0 = \frac{\varepsilon_0 (1 - LC\omega^2)}{[(R - LRC\omega^2)^2 + (\omega L)^2]^{1/2}} = \frac{\varepsilon_0}{\sqrt{R^2 + \frac{L^2 / C^2}{(\omega L - \frac{1}{\omega C})^2}}} = \frac{\varepsilon_0}{Z}$$

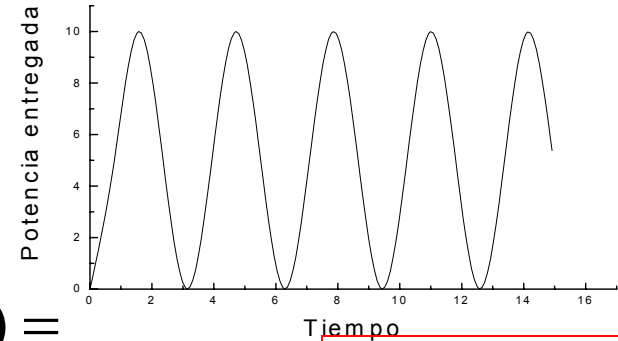
$$\text{si } \omega = \frac{1}{\sqrt{LC}} \Rightarrow i = 0$$

**No hay caída en R y toda la tensión en //**

## Potencia en circuitos de alterna

$$\varepsilon = \varepsilon_0 \text{ sen } \omega t \quad i = i_0 \text{ sen } (\omega t + \varphi)$$

$$P(t) = i \varepsilon = i_0 \varepsilon_0 \text{ sen } \omega t \text{ sen } (\omega t + \varphi)$$



**Función impar**

## Sentido de potencia instantánea?

$$\bar{P} = \frac{1}{T} \int_0^T i \varepsilon dt = \frac{1}{T} \int_0^T i_0 \varepsilon_0 \text{ sen } \omega t \text{ sen } (\omega t + \varphi) dt = \frac{i_0 \varepsilon_0}{T} \left( \int_0^T \text{sen } \omega t \text{ sen } \omega t \cos \varphi + \int_0^T \text{sen } \omega t \cos \omega t \text{ sen } \varphi \right)$$

$$\bar{P} = \frac{i_0 \varepsilon_0}{2} \cos \varphi$$

$$i_{ef} = i_0 / \sqrt{2}$$

$$\varepsilon_{ef} = \varepsilon_0 / \sqrt{2}$$

**Valores constantes de i y V que producen la misma potencia media que la tensión de alterna**

**220 V de línea es Vef: 310 de pico**

**cos φ Factor de potencia**

**Al circuito inductivo o capacitivo (φ ± π/2) no se le entrega potencia**

$$P_a = i_{ef} \varepsilon_{ef} \cos \varphi \quad \text{Potencia activa (la que se disipa en R)} \quad i_{ef} \varepsilon_{ef} \frac{R}{Z} = i_{ef}^2 R$$

$$P_r = i_{ef} \varepsilon_{ef} \text{ sen } \varphi \quad \text{Potencia reactiva (la que oscila)}$$

$$P_{ap} = i_{ef} \varepsilon_{ef} \quad \text{Potencia aparente}$$

**Para una P requerida, a > cos φ < i. Multas por bajo cos φ. Se cobre VA**

# Tratamiento con números complejos

Elementos en serie  $V(t) = \sum V_k(t)$  Elementos en paralelo  $i(t) = \sum i_k(t)$

Se definen tensión y corriente complejas

$$V_0 \neq \sum V_{k0}$$

$$i_0 \neq \sum i_{k0}$$

$$E = \varepsilon_0 e^{j\varphi_1} = \varepsilon_0 \cos \varphi_1 + j \varepsilon_0 \operatorname{sen} \varphi_1 = a + jb$$

$$|E| = \varepsilon_0 = \sqrt{a^2 + b^2} \quad \varphi_1 = \operatorname{arctg} \frac{b}{a} \quad V = \operatorname{Re}(E e^{j\omega t})$$

$$I = i_0 e^{j\varphi_2} = i_0 \cos \varphi_2 + j i_0 \operatorname{sen} \varphi_2 = c + jd$$

$$|I| = i_0 = \sqrt{c^2 + d^2} \quad \varphi_2 = \operatorname{arctg}(c/d) \quad i = \operatorname{Re}(I e^{j\omega t})$$

$$i_1 + i_2 = \operatorname{Re}[(I_1 + I_2) e^{j\omega t}]$$

Si  $\varepsilon = \varepsilon_0 \cos \omega t$

En R  $I = \frac{\varepsilon_0}{R} e^{j\omega t} \quad I = \varepsilon_0 / R$

En C  $I = \varepsilon_0 \omega C e^{j(\pi/2)} \quad I = j \varepsilon_0 \omega C$

En L  $I = \frac{\varepsilon_0}{\omega L} e^{-j(\pi/2)} \quad I = -j(\varepsilon_0 / \omega L)$

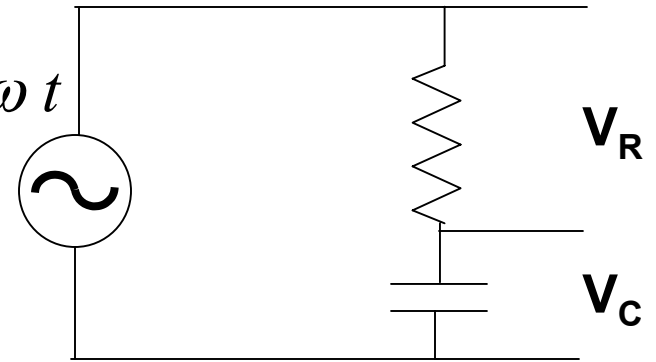
	Z	Y	Z*	Y*
R	R	1/R	R	1/R
C	1/ωC	ωC	$\frac{-j}{\omega C}$	jωC
L	ωL	$\frac{1}{\omega L}$	jωL	$\frac{-j}{\omega L}$

## Pasa altos y pasa bajos

$$\varepsilon = \varepsilon_0 \text{ sen } \omega t$$

$$i = i_0 \text{ sen } (\omega t + \varphi) \quad \varepsilon_0 = i_0 \sqrt{R^2 + (1/\omega C)^2}$$

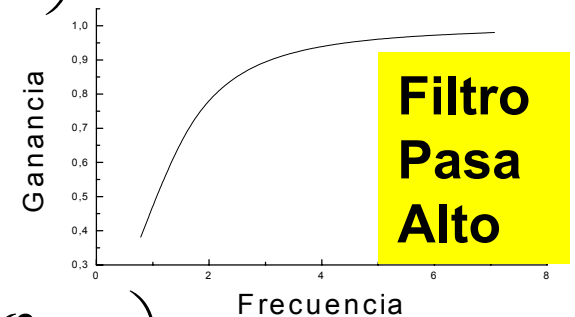
$$\text{tg } \varphi = \frac{1/\omega C}{R} \quad q = \int i dt = -\frac{i_0}{\omega} \cos(\omega t + \varphi)$$



$$\Delta V_R = iR = R(\varepsilon_0/Z) \text{ sen } (\omega t + \varphi) \quad \Delta V_C = q/C = -(\varepsilon_0/Z \omega C) \cos(\omega t + \varphi)$$

$$\frac{\bar{V}_R}{\varepsilon} = \frac{(\varepsilon_0 R/Z)}{\tau \varepsilon_0} \left( \int_0^\tau \frac{\text{sen } \omega t \cos \varphi}{\text{sen } \omega t} dt + \int_0^\tau \cancel{\cot g \omega t \text{ sen } \varphi} dt \right)$$

$$\frac{\bar{V}_R}{\varepsilon} = \frac{R}{Z} \cos \varphi = \frac{R^2}{R^2 + (1/\omega C)^2} \rightarrow \begin{matrix} 0 & \text{si } \omega \rightarrow 0 \\ 1 & \text{si } \omega \rightarrow \infty \end{matrix}$$



$$\frac{\bar{V}_C}{\varepsilon} = \frac{-(\varepsilon_0/Z \omega C)}{\tau \varepsilon_0} \left( \int_0^\tau \cancel{\cos \omega t \cos \varphi} dt - \int_0^\tau \frac{\text{sen } \omega t \text{ sen } \varphi}{\text{sen } \omega t} dt \right)$$

$$\frac{\bar{V}_C}{\varepsilon} = \frac{1/\omega C}{Z} \text{ sen } \varphi = \frac{(1/\omega C)^2}{R^2 + (1/\omega C)^2} \rightarrow \begin{matrix} 1 & \text{si } \omega \rightarrow 0 \\ 0 & \text{si } \omega \rightarrow \infty \end{matrix}$$

